

Implications of a Dilaton in Gauge Theory and Cosmology

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Abstract: After a review of theoretical motivations to consider theories with direct couplings of scalar fields to Ricci and gauge curvature terms, we consider the dynamics and non-perturbative stabilization of a dilaton in three and in four dimensions. In particular, we derive generalized Coulomb potentials in the presence of a dilaton and discuss a low energy effective dilaton potential induced by instanton effects and the S-dual coupling to axions. We conclude with a discussion of cosmological implications of a light dilaton.

1 Introduction

It is a widespread belief both in elementary particle physics and in general relativity that theoretical Ansätze for physics at the Planck scale are lacking experimental relevance today and in the foreseeable future. It is indeed a generic feature of theories unifying gravity and quantum field theory to make predictions mostly for physics at the Planck scale, and eventually also for a GUT scale a few orders of magnitude below the Planck scale. However, many seriously pursued proposals for a framework of quantum gravity, including string theory, predict scalar particles which couple both to gravity and matter fields. These scalar particles must be very weakly coupled or have to acquire large masses in order to meet experimental and cosmological constraints, or there must exist other non-perturbative mechanisms to make these scalars invisible in the low energy regime.

A particularly interesting deviation from standard Einstein–Yang–Mills theory is the prediction of a dilaton. There is no unique definition of dilatons, but a common feature of dilatons appearing in different theoretical frameworks is their direct coupling to Ricci or gauge curvature terms. A typical feature of the dilaton in string theory is its exponential coupling to Yang–Mills terms in the frame where the dilaton decouples from the curvature scalar R , and emphasis in the present paper will be on a scalar field ϕ which couples to gravity and gauge fields in four dimensions through

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2\kappa}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi \cdot \partial_\nu\phi - \frac{1}{4}\exp(\frac{\phi}{f_\phi})F_{\mu\nu}^j F^{\mu\nu}_j,$$

where the mass scale f_ϕ characterizes the strength of the coupling. Following usual conventions we denote the coupling scale f_ϕ also as a decay constant, since in many formulas it appears similar to a decay constant of a scalar bound state. However, there is an important difference: The dilatation current containing a piece $f_\phi\partial_\mu\phi$ does not parametrize a physical decay amplitude of fundamental string or Kaluza–Klein dilatons, and f_ϕ enters only with negative powers in actual transition amplitudes.

In a different setting fundamental scalars are introduced with a direct coupling to the curvature scalar to mimic a time dependent gravitational constant, to serve as an additional gravitational degree of freedom, or for the sake of local scale invariance. A scalar coupling both to the Ricci scalar and to the Maxwell term was investigated for a long time by Jordan and his collaborators. Jordan’s motivation originated from Dirac’s proposal of a variable gravitational constant [40], and from the observation that Kaluza–Klein theory supports Dirac’s proposal if the four-dimensional metric is directly induced from a five-dimensional metric without rescaling. The coupling of the scalar to a Maxwell term is then a direct consequence of Kaluza–Klein theory. However, later Jordan abandoned the coupling to electrodynamics, thus anticipating a particular case of a Brans–Dicke theory of gravity [61].

Motivated from Mach’s principle Brans and Dicke introduced a scalar $\Phi_{\mathcal{BD}}$ coupling through $R\Phi_{\mathcal{BD}}$ to account for a long range scalar field participating in the gravitational interaction [16]. In order to maintain the property that gravitational fields can be gauged away locally, they required from the outset ordinary metric couplings of all matter fields,

thus excluding in particular the coupling of the scalar to gauge fields. More general scalar-tensor theories of gravity have later been defined as theories which can be related to a Brans–Dicke theory through a $\Phi_{\mathcal{BD}}$ -dependent rescaling of the metric [102], but due to the Weyl invariance of Yang–Mills terms in four dimensions these theories also contain no direct coupling to the Yang–Mills curvature.

In spite of our emphasis on couplings to Yang–Mills terms, scalar-tensor theories will be of some interest to us, due to a theoretical ambiguity in low energy string theory: It is known since long that the low energy and low curvature limit of string theory in the critical dimension is given by Einstein gravity, and this property persists in lower dimensions if low-dimensional metrics are embedded appropriately in higher-dimensional metrics. However, the claim that the leading curvature term in four-dimensional string effective actions is given by the Einstein–Hilbert term was challenged recently by Gasperini and Veneziano, see [50, 49] and references there. A compactification which rescales the low-dimensional metric by the string dilaton instead of a Kaluza–Klein dilaton gives a Brans–Dicke type theory in the curvature sector, and the problem whether low energy gravity in string theory is described by Einstein gravity in the Einstein frame or by a scalar-tensor theory in the string frame is an experimental issue.

The gravitational sector of a scalar-tensor theory with constant Brans–Dicke parameter $\omega_{\mathcal{BD}}$ reads

$$\begin{aligned}\mathcal{L}_{\mathcal{BD}} &= \frac{1}{2}\sqrt{-g}\Phi_{\mathcal{BD}}\left(R - \omega_{\mathcal{BD}}g^{\mu\nu}\partial_\mu\ln(\Phi_{\mathcal{BD}})\cdot\partial_\nu\ln(\Phi_{\mathcal{BD}}) + 2\Lambda(\Phi_{\mathcal{BD}})\right) \\ &= \sqrt{-g}\left(\frac{1}{8|\omega_{\mathcal{BD}}|}\phi^2R - \text{sgn}(\omega_{\mathcal{BD}})\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\cdot\partial_\nu\phi + \Lambda\left(\frac{\phi^2}{4|\omega_{\mathcal{BD}}|}\right)\frac{\phi^2}{4|\omega_{\mathcal{BD}}|}\right).\end{aligned}\tag{1}$$

We have to be careful with the sign of $\omega_{\mathcal{BD}}$ since the Brans–Dicke parameter can attain negative values. We will see in section 3 that a “strong” version of Kaluza–Klein theory yields Brans–Dicke parameters $0 > \omega_{\mathcal{BD}} > -1$, and string theory in the string frame has $\omega_{\mathcal{BD}} = -1$.

A Brans–Dicke type coupling of a scalar field also emerged in constructions of locally Weyl invariant theories without a Weyl vector, see [105, 118, 42, 41] and references there. Coupling of a massless scalar ϕ to other particle masses through the substitution

$$m \rightarrow m\phi$$

ensures Weyl invariance with standard kinetic terms if the rescaling of the metric is accompanied by appropriate rescalings of scalar and spinor fields. Then local scale invariance can be ensured to first order if $\partial_\mu\ln\phi$ is used as a connection and if the kinetic term for ϕ corresponds to a Brans–Dicke theory in the singular limit $\omega_{\mathcal{BD}} \rightarrow -\frac{3}{2}$.

Both in scalar-tensor theories and in Weyl invariant theories the dilaton ϕ does not couple to Yang–Mills terms. However, it does couple both to Ricci and gauge curvature terms in string theory in the string frame and in Kaluza–Klein theory without metric rescaling, whereas it decouples from the Ricci scalar in Weyl transformed Kaluza–Klein theory and

in string theory in the Einstein frame:

Theory	Curvature term	Yang–Mills term	Mass terms
String theory, string frame	$\phi^2 R$	$\phi^2 F^2$	m
String theory, Einstein frame	R	$\exp(\phi/f_\phi) F^2$	m
Brans–Dicke theory	$\phi^2 R$	F^2	m
Weyl invariant theory	$\phi^2 R$	F^2	$m\phi$

Table 1: Theories with fundamental scalars coupling to curvature terms.

In the normalization of these terms a canonical kinetic term for the scalar ϕ was assumed. The “strong” and “weak” versions of Kaluza–Klein theory are related to string theory in the string frame and in the Einstein frame, respectively. In neglecting mass dependent couplings of the dilaton in the string and Kaluza–Klein framework we assumed that the mass terms originate at a scale far below any string or compactification scale. Otherwise couplings to mass terms would also appear in this sector, see section 3.

Generically different rows in this table can be connected through field dependent rescalings of the metric. This can be a useful mathematical tool in analyzing e.g. equations of motion. However, it must be stressed that field dependent rescalings of metrics are not symmetry transformations of one and the same physical system, but generically have different physical implications.

It must also be emphasized that Table 1 is by no means exhaustive: A theory may contain several dilatonic degrees of freedom, and a particularly important example is compactified heterotic string theory: This contains besides the model independent string dilaton ϕ_S at least one Kaluza–Klein dilaton ϕ_K , and a linear combination ϕ of the two couples to four-dimensional gauge fields, whereas in non-rescaled compactification or in the string frame ϕ_K or ϕ couple to R , respectively, while no dilaton couples to R in the Einstein frame. Furthermore, in recent years string theory motivated discussions of more complicated coupling functions both in the curvature and in the Yang–Mills sector.

Couplings of light scalars to Ricci or Yang–Mills curvature have very interesting cosmological implications, and in particular a direct coupling of a scalar to the Einstein–Hilbert term can have drastic consequences like removing the initial singularity of space–time without a string threshold. Cosmological implications of a Brans–Dicke scalar with and without a cosmological function Λ were discussed in [108]. String theory in the string frame has a dilaton with a Brans–Dicke type coupling, and the work of Veneziano and his collaborators attracted much interest in the resulting Brans–Dicke type cosmology known as string cosmology [50]. The cosmological implications of a dilaton in Weyl invariant theories were investigated in [78, 21]. For a discussion of a low energy string dilaton in the Einstein frame, see e.g. [34, 35]. There a duality invariant coupling between the dilaton and the axion played a crucial role in identifying a mass generating mechanism for the dilaton. However, a possible cosmological significance of axion–dilaton interactions was emphasized already in [20],

where the common appearance of axions and dilatons was motivated from supersymmetry [112, 31].

Superstring theory currently undergoes a major change of paradigm: Numerous duality symmetries have been established or proposed between seemingly different versions of the theory in various macroscopic dimensions, and it has become particularly clear that symmetry transformations interchanging axions and dilatons play a significant and fundamental role in the theory. These symmetries go by the name strong/weak coupling duality, or S-duality for short, because they involve sign flips of the dilaton. Since the expectation value of the exponentiated dilaton determines the strength of the string coupling, those sign flips can interchange strongly coupled regimes of string theory with weak coupling regimes.

We will see that under a certain constraint on decay constants of the axion and the dilaton to be explained in section 6, a low energy imprint of S-duality can still mix the dilaton and the axion. This indicates in particular that a fundamental light axion should come with a dilaton, and that their properties should be intimately connected. Now it is known since long that the gauge theory of strong interactions [48] strongly motivates considerations of a light axion, and that instantons create an effective potential for this axion. If the QCD axion is a fundamental pseudo-scalar, and if axion-dilaton duality is realized at some scale, then we should also expect a fundamental dilaton coupling to QCD. The main motivation for the present work was the observation that instantons induce an interesting potential for such a dilaton far below the scale of supersymmetry breaking. Such a possibility was not pursued before, since an estimate on the coupling of a QCD dilaton to nucleons seems to violate the Newton approximation for weak gravitational fields, see [43] and references there:

Constraints on dilaton masses without a direct coupling to the Ricci scalar are based on the assumption that the dilaton can be treated as a local gauge coupling which shows up in nucleon masses. This might imply a coupling of the dilaton to hadronic mass terms in low energy effective theories, eventually also inducing material dependent couplings to macroscopic bodies. An estimate derived on this basis states that the dilaton should be heavier than 10^{-4} eV to be on the safe side, too heavy for a QCD dilaton with a coupling scale f_ϕ of the order of the Planck mass¹. The reasoning implied in the derivation of this estimate was ingenious, but a weak point concerns the interpretation of the dilaton as a local gauge coupling: We will calculate the impact of a dilaton on the classical $1/r$ interaction of gauge charges in section 5 and find that the dilaton either regularizes the Coulomb potential at a distance $r_\phi \sim f_\phi^{-1}$ or implies confinement, with an interaction potential between stationary charges raising linearly with the distance. These results indicate that the impact of a dilaton in gauge theory is not adequately approximated by an effective local gauge coupling, and it seems premature to conclude that the dilaton-gluon coupling induces a dilaton-nucleon coupling proportional to the mass terms in low energy hadron theories. The problem in how far such a coupling would amount to material dependent couplings also causes some uncertainty, and lacking a reliable quantitative picture of the emergence

¹Our conventions for Planck units are $m_{Pl} = (8\pi G)^{-1/2} = 2.4 \times 10^{18}$ GeV, $t_{Pl} = 2.7 \times 10^{-43}$ s, $l_{Pl} = 8.1 \times 10^{-35}$ m.

of the hadron spectrum in QCD both with and without a dilaton, we are currently unable to discuss constraints from the weak equivalence principle on macroscopic bodies. On the other hand, constraints from elementary particle masses in the standard model do not arise, since the dilaton originates at string or Kaluza–Klein scales far beyond the weak scale, and therefore neither the string dilaton nor a Kaluza–Klein dilaton are expected to couple to mass terms in the standard model. However, in section 8 we will also discuss implications of a dilaton with a mass $m_\phi \geq 10^{-4}$ eV. If the mechanism of axion induced dilaton stabilization outlined in section 6 is correct this mass would correspond to a decay constant $f_\phi \leq 10^{11}$ GeV far below the Planck scale, but it would still be invisible from a particle physics point of view and have interesting cosmological implications. Of course, from a stringy point of view such a low decay constant is a puzzle, since string theory generically predicts dilaton coupling scales of the order of the Planck mass.

While the results of section 5 show that the dilaton in gauge theory can not be addressed as an effective local gauge coupling, the expectation value of the dilaton still seems to imply an ambiguity in the definition of the coupling. To elucidate this consider the Lagrangian (in flat Minkowski space)

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi \cdot \partial^\mu\phi - \frac{1}{4}\exp\left(\frac{\phi}{f_\phi}\right)F_{\mu\nu}^j F^{\mu\nu}_j + \bar{\psi}(i\gamma^\mu\partial_\mu + q\gamma^\mu A_\mu - m)\psi$$

describing gauge theory with a dilaton ϕ , gauge coupling q and fermions ψ . For fixed ψ the mapping

$$\begin{aligned} q &\rightarrow q' \\ A_\mu &\rightarrow A'_\mu = \frac{q}{q'}A_\mu \\ \phi &\rightarrow \phi' = \phi - 2f_\phi \ln\left(\frac{q}{q'}\right) \end{aligned}$$

leaves the Lagrangian invariant but implies e.g. a rescaling of the 1–gluon exchange 4–fermion amplitude $\mathcal{A}_{2\rightarrow 2} \rightarrow \frac{q'^2}{q^2}\mathcal{A}_{2\rightarrow 2}$. This is the gauge theory version of the problem of the running dilaton or of the constancy of constants (see e.g. [39]): Motion of the expectation value of the dilaton implies a rescaling of the gluon propagator and a corresponding rescaling of the gauge coupling measured in scattering events.

On the other hand, at least the variation of the electromagnetic fine structure constant $\alpha_{em} = \frac{e^2}{4\pi}$ over cosmic time scales is strongly constrained: Improving on an analysis by Shlyakhter [89], Damour and Dyson point out that the fine structure constant about two billion years ago, when the natural Oklo fission reactor in West Africa was active, differs from todays value at most by $\frac{|\delta\alpha|}{\alpha} < 1.2 \times 10^{-7}$ [26]. On larger time scales, Varshalovich et al. give $\frac{|\delta\alpha|}{\alpha} < 1.6 \times 10^{-4}$ for the variation of the fine structure constant between ultraviolet emission of high red shift ($z \sim 3$) quasars and today [99]. In a flat universe electromagnetic waves emitted at $z = 3$ travelled for almost 88% of the lifetime of the universe (i.e. they were emitted certainly more than 9 billion years ago), and if a dilaton also couples to the photon this means that some mechanism must have stabilized the expectation value of the dilaton at an early stage in the evolution of the universe.

Usual attempts to solve the stability problem for the dilaton link the generation of a low energy dilaton potential to gaugino condensation: Due to the dilaton gaugino coupling implied by supersymmetry, a gaugino condensate provides an attractive mechanism to generate a dilaton mass. However, it must be pointed out that a gaugino condensate in the most direct and simple supersymmetric Yang–Mills dilaton theory would send the dilaton expectation value to $-\infty$ rather than stabilizing it at some finite value. This is a simple consequence of the fact that the dilaton couples to gluinos with an exponential $\exp(2\phi/f_\phi)$, i.e. with the same sign as in the dilaton gluon coupling. Therefore anomalous low energy effective terms [101, 94] or duality invariant potentials have to be employed to stabilize dilatonic degrees of freedom, see e.g. [46, 75] and references there.

The puzzle can be resolved in a different way by noting that the dilaton couples with a term $\exp(-2\phi/f_\phi)$ to the kinetic energy of the axion, whence a non-vanishing variance of the axion would provide a direct and simple way to generate a dilaton mass [34, 35]. This mechanism works with or without a dilaton gluino coupling, and the discussion in section 6 concentrates on the non-supersymmetric case.

In a string inspired four-dimensional field theory the dilaton coupling to gauge fields will generically be a superposition of the massless closed string excitation accompanying the graviton and a Kaluza–Klein dilaton defined as a logarithm of the determinant of the internal metric. In compactifications of heterotic string theory, e.g., the four-dimensional dilaton is dominated by a Kaluza–Klein component with a mixing angle of 30° into the string dilaton.

Like the string dilaton the Kaluza–Klein dilaton comes initially without a mass. This initial absence of a dilaton potential in the low energy sector is easy to understand: Since the low energy effective fields are zero modes of the internal derivative operators, they couple to the fluctuations of the internal dimensions, but carry no remembrance of their actual size. Therefore, we may expect dilaton stabilization in the low energy sector only if there exist non-perturbative effects which are genuinely related to the number of macroscopic dimensions, since these effects would have to disappear upon decompactification or further compactification. We know such a genuinely four-dimensional non-perturbative effect very well: Instantons in gauge theories may provide a mechanism to stabilize four dimensions, i.e. we suspect that instantons induce an effective dilaton potential in the low energy regime. However, suppose we start with some large gauge group G which decomposes into several abelian and non-abelian factors: Where do we expect the dominant instanton contributions? Instanton energies go with the inverse gauge coupling squared, but instantons are suppressed in broken gauge groups due to the large masses of the gauge fields. Therefore the dominant instanton contributions should arise in that factor of G which corresponds to the non-abelian unbroken symmetry with the largest coupling constant. If instantons stabilize the dilaton, it is QCD which has to make the dominant contribution!

Of course, there may appear other non-perturbative effects in four dimensions, eventually breaking supersymmetry and active at a much higher scale. It is well conceivable that such effects may also create an effective dilaton potential, and in a sense this is a working hypothesis for mainstream research in the field. I make no attempt to invalidate the

mainstream approach which links the dilaton potential to supersymmetry breaking, but I suppose there is enough motivation to consider an axion–dilaton system which is stabilized at or above the QCD phase transition, around 10^{-4} seconds after the big bang.

We have defined the dilaton in four dimensions through its characteristic coupling to gauge fields, yet we have to re–address its coupling to gravity, in order to explain why we exclude such couplings in the present paper: The dilaton can couple to the Einstein–Hilbert term through a term $U(\phi)R$, and we have pointed out already that direct compactification of dimensions would predict a polynomial coupling of Kaluza–Klein dilatons both to R and to gauge fields. In the gravitational sector this would correspond to a Brans–Dicke type theory of gravity with a Brans–Dicke parameter $\omega_{\mathcal{BD}} \simeq -1$ (see section 3). However, through appropriate Weyl rescalings the dilaton can be arranged to couple to gravity in a standard way without coupling directly to the curvature scalar R . The set of fields and the metric where the lowest order gravitational action is given by the Einstein–Hilbert term $\int d^4x \sqrt{-g}R$ is the Einstein frame, and we will mainly use this frame. This is not just a matter of taste, but chosen on the basis of experimental constraints: Solar system measurements of light bending and time delay in gravitational fields are known to constrain deviations from standard Einstein gravity to less than 10^{-3} , and this imposes rather strong limits on scalar–tensor theories of gravity. It implies in particular that the inverse Brans–Dicke parameter measuring the direct coupling of light scalar fields to curvature is very small, implying in turn that even if the physical metric would not correspond to an Einstein frame, it would have to be very close to an Einstein frame. It was also mentioned before that the string dilaton does not couple to the Einstein–Hilbert term in the critical dimensions 10 and 26 [53, 18, 69].

There are other viable alternatives than directly relying on a Weyl transformed metric as the physical metric: If the Brans–Dicke scalar somehow acquires such a large mass that it effectively is frozen to a constant value, this would certainly comply with all contemporary tests of Einstein gravity. Furthermore, Damour and Nordtvedt pointed out that a scalar–tensor theory of gravity with a convex coupling function $U(\phi)$ also effectively reduces to Einstein gravity, even without a mass term [27].

In order to facilitate the comparison with calculations in other frames, and to clarify which models are related through Weyl rescalings, it is useful to have a dictionary of the behavior of various sectors in the Lagrangian under Weyl transformations: Under rescalings of the metric in D dimensions

$$\tilde{g}_{\mu\nu} = \exp\left(2\frac{\lambda - \tilde{\lambda}}{D-2}\phi\right)g_{\mu\nu}$$

the gravitational sector transforms according to

$$\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\exp(\tilde{\lambda}\phi)(\tilde{R}_{\mu\nu} + \frac{D-1}{D-2}\tilde{\lambda}^2\partial_\mu\phi\cdot\partial_\nu\phi) = \sqrt{-g}g^{\mu\nu}\exp(\lambda\phi)(R_{\mu\nu} + \frac{D-1}{D-2}\lambda^2\partial_\mu\phi\cdot\partial_\nu\phi).$$

In the matter sector one finds for Yang–Mills fields

$$\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\tilde{g}^{\rho\sigma}\exp(\tilde{\alpha}\phi)\text{tr}(F_{\mu\rho}\cdot F_{\nu\sigma}) = \sqrt{-g}g^{\mu\nu}g^{\rho\sigma}\exp(\alpha\phi)\text{tr}(F_{\mu\rho}\cdot F_{\nu\sigma})$$

with

$$\alpha = \tilde{\alpha} + \frac{D-4}{D-2}(\lambda - \tilde{\lambda}),$$

while fermions coupling to Yang–Mills fields A and a canonical spin connection Ω transform according to

$$\sqrt{-\tilde{g}}\tilde{\psi}[\tilde{e}^\mu{}_a\gamma^a(\partial_\mu + \tilde{\Omega}_\mu - iqA_\mu) + im]\tilde{\psi} = \sqrt{-g}\bar{\psi}[e^\mu{}_a\gamma^a(\partial_\mu + \Omega_\mu - iqA_\mu) + \exp\left(\frac{\lambda - \tilde{\lambda}}{D-2}\phi\right)im]\psi$$

with

$$\begin{aligned}\tilde{\psi} &= \exp\left(-\frac{D-1}{D-2}\frac{\lambda - \tilde{\lambda}}{2}\phi\right)\psi, \\ \tilde{\Omega}^a{}_{b\mu} &= \Omega^a{}_{b\mu} + \frac{\lambda - \tilde{\lambda}}{D-2}(e_\mu{}^a e^\nu{}_b - e_{\mu b} e^{\nu a})\partial_\nu\phi.\end{aligned}$$

As an application, the combined Weyl transformation and redefinition

$$\begin{aligned}g_{\mu\nu} &= F(\Phi)^{\frac{2}{D-2}}\tilde{g}_{\mu\nu} \\ \phi &= \int d\Phi \sqrt{\frac{G(\Phi)}{F(\Phi)} + \frac{D-1}{(D-2)\kappa} \frac{F'(\Phi)^2}{F(\Phi)^2}}\end{aligned}$$

transform Jordan–Brans–Dicke type theories into the (physically inequivalent) Einstein frame:

$$\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\left(\frac{1}{2\kappa}F(\Phi)\tilde{R}_{\mu\nu} - \frac{1}{2}G(\Phi)\partial_\mu\Phi \cdot \partial_\nu\Phi\right) = \sqrt{-g}g^{\mu\nu}\left(\frac{1}{2\kappa}R_{\mu\nu} - \frac{1}{2}\partial_\mu\phi \cdot \partial_\nu\phi\right),$$

and according to the previous paragraph we may also transform the kinetic terms of the fermions to standard form.

Our analysis in four dimensions will be mainly based on gauge theory coupled to an axion a and a dilaton ϕ in the Einstein frame:

$$\begin{aligned}\frac{1}{\sqrt{-g}}\mathcal{L} &= \frac{1}{2\kappa}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi \cdot \partial_\nu\phi - \frac{1}{2}\exp\left(-2\frac{\phi}{f_\phi}\right)g^{\mu\nu}\partial_\mu a \cdot \partial_\nu a \\ &\quad - \frac{1}{4}\exp\left(\frac{\phi}{f_\phi}\right)F_{\mu\nu}{}^j F^{\mu\nu}{}_j + \frac{q^2}{64\pi^2 f_a}\epsilon^{\mu\nu\rho\sigma}a F_{\mu\nu}{}^j F_{\rho\sigma j}.\end{aligned}\tag{2}$$

Superficially, we will refer to this system as an axion–dilaton–gluon system, irrespective of whether the gauge group is $SU(3)$ or some other Lie group.

The particular ratio between the couplings of the dilaton to gauge fields and the axion in (2) can be motivated for two reasons: On the one hand, this ratio of couplings arises automatically in Kaluza–Klein compactifications to four dimensions, as will be elucidated in section 3, while on the other hand the same ratio also arises from the requirement of duality symmetry in the axion–dilaton system. The last, and maybe most important motivation for consideration of the axion–dilaton–gluon system comes from the observation

that this system must hold a clue to the issue of dilaton stabilization in four dimensions: It is known that instantons create an effective axion potential $V(a) \sim -\cos(a/f)$ plus higher order terms in $\cos(a/f)$. We will see that instantons and the effective axion potential survive introduction of the dilaton and break the scale invariance of the equations of motion following from (2). This provides a strong hint that instantons and axions must also lift the degeneracy of the dilaton potential associated with the scale invariance of the tree level equations of motion.

The resulting dilaton potential has many interesting implications on the low energy sector of string theory and cosmology. In particular, the dynamics of the dilaton switches from expansion dominance to an oscillatory behavior around 10^{-4} seconds after the initial singularity, that is “long” after the onset of axion oscillations ($\sim 10^{-6}$ seconds). At this time the temperature of the universe has already dropped to a value near the QCD phase transition, and the dilaton can make an appreciable contribution to the energy density of the universe as a cold dark matter candidate if its variance above 1 TeV is of the order of the Planck mass $\sqrt{\phi^2} \sim m_{Pl}$.

Investigation of the dilaton potential resulting from the observations outlined above and discussions of cosmological implications will be a primary concern in the present work, and we will concentrate on these tasks especially in sections 6–8. Besides this, we will discuss stabilization of the dilaton in three dimensions in section 4, while the impact of the dilaton on potentials of pointlike particles in four-dimensional gauge theory is examined in section 5. Sections 2 and 3 survey introductory material needed in the discussion of the dilaton potential. While no attempt was made to make the paper fully self-contained, the introductory sections should make it amenable to non-experts with some basic knowledge in quantum field theory, string theory and cosmology.

2 Instantons in gauge theory

Instantons are classical solutions of Yang–Mills equations on Euclidean four-dimensional manifolds [12, 59, 5]. They play a prominent role in particle physics through their contribution to 't Hooft's solution of the strong U(1) problem and through their contribution to chiral symmetry breaking: Chiral symmetry breaking is signaled through a quark condensate which can be related to the spectral density $\rho(\lambda)$ of the Euclidean Dirac operator through a celebrated relation of Banks and Casher [7]

$$\langle \bar{q}q \rangle = -\pi\rho(0).$$

While we are still lacking a full understanding of the dynamics of chiral symmetry breaking in QCD from first principles, the emergence of a non-vanishing spectral density at eigenvalue zero can be attributed to the presence of an instanton liquid in the Euclidean vacuum, as has been thoroughly reviewed in [82, 91].

In the present setting we are interested in instantons because some appropriate generalization of them will contribute to the effective potential of the dilaton, and we will review some of the properties of instantons in this section. Classical and very thorough reviews of instantons have been given in [23, 98]. For the ADHM construction of the general multi-instanton solution see [5] and [24] and references there. The path breaking work on the calculation of quantum effects was [59].

In this introductory section we will only review the one-instanton solution elaborating on a theorem of Wilczek. Wilczek's theorem relates instantons to conformally flat spaces of constant scalar curvature [109, 19, 32] and has the virtue to naturally yield 't Hooft's Ansatz and to explain the group theoretic origin of the 't Hooft symbols [59].

Throughout this section we will work with two conformally related Euclidean 4-spaces, one of those being flat while the other metric can be written as

$$g_{\mu\nu}(x) = \chi^2(x)\delta_{\mu\nu} = \frac{1}{\sigma^2(x)}\delta_{\mu\nu}.$$

Indices raised with $g^{\mu\nu}(x)$ will be denoted by a dot in this section: $g^{\mu\nu}(x)v_\nu(x) = v^\dot{\mu}(x)$. The gauge covariant derivative is² $D_\mu = \partial_\mu + \mathcal{A}_\mu$ and the covariant derivative in the conformally flat space is ∇_μ . Since we will not perform Legendre transformations in the Euclidean setting and the Euclidean partition function resembles a canonical ensemble, we will use the terms action and energy synonymously in this section.

The well-known local isomorphism $\text{SO}(4) \cong \text{SU}(2) \times \text{SU}(2)$ may be used to reduce $\text{gl}(4, \mathbb{R})$ connections Γ to $\text{su}(2)$ connections ${}^\pm\mathcal{A}$ according to

$${}^\pm\mathcal{A}_{\mu i} = -({}^\pm Z_i)^\alpha{}_\beta \Gamma^\beta{}_{\alpha\mu}, \quad (3)$$

²We are using an anti-hermitian basis for \mathcal{A} . The relation between the gauge potential $\mathcal{A}_\mu \equiv A_{\mu i}(-iX^i)$ and $A_\mu \equiv A_{\mu i}X^i$ is $\mathcal{A}_{\mu i} = qA_{\mu i}$.

where an explicit representation of the $SU(2)$ projectors is

$$(\pm Z_i)_{\mu\nu} = \frac{1}{2}(\pm \delta^0_{\nu} \delta_{i\mu} \mp \delta^0_{\mu} \delta_{i\nu} - \epsilon_{ijk} \delta^j_{\mu} \delta^k_{\nu}).$$

These projectors provide a selfdual and an anti-selfdual basis for four-dimensional representations of $su(2)$, and in compliance with the uniqueness of the $\text{spin}-\frac{3}{2}$ equivalence class, they are intertwined via $\mathbb{T} \cdot {}^+Z_i \cdot \mathbb{T} = {}^-Z_i$ with $\mathbb{T} = \text{diag}\{-1, 1, 1, 1\}$. Identifying indices which carry the values 0 or 4, these generators are related to the 't Hooft symbols via

$$({}^+Z_i)_{\mu\nu} = -\frac{1}{2}\bar{\eta}_{i\mu\nu}, \quad ({}^-Z_i)_{\mu\nu} = -\frac{1}{2}\eta_{i\mu\nu}.$$

Thus the appendix of [59] may be used if some signs are adjusted properly due to $\epsilon_{0123} = -\epsilon_{1234}$. The reduction according to (3) yields $su(2)$ -valued curvatures

$$\pm \mathcal{F}_{\mu\nu}{}^i = -(\pm Z^i)^\alpha{}_\beta (R^\beta{}_{\alpha\mu\nu} + S^\beta{}_{\alpha\mu\nu}) \quad (4)$$

with the deviation from the Riemannian

$$S^\alpha{}_{\beta\mu\nu} = \frac{1}{2}\Gamma^\rho{}_{\beta\mu}(\Gamma^\alpha{}_{\rho\nu} + \Gamma_\rho{}^\alpha{}_\nu) - \frac{1}{2}\Gamma^\alpha{}_{\rho\mu}(\Gamma^\rho{}_{\beta\nu} + \Gamma_\beta{}^\rho{}_\nu).$$

The conformal Ansatz is now introduced by inserting the Levi-Civita connection

$$\Gamma^\rho{}_{\mu\nu} = \frac{1}{2\sigma^2}(\delta_{\mu\nu}\partial^\rho\sigma^2 - \delta^\rho{}_\mu\partial_\nu\sigma^2 - \delta^\rho{}_\nu\partial_\mu\sigma^2) \quad (5)$$

corresponding to the conformally flat metric g . This Ansatz leads to $S = 0$, and (3) reduces to the 't Hooft Ansatz:

$$\pm \mathcal{A}_{\mu i} = (\pm Z_i)^\rho{}_\mu \partial_\rho \ln(\sigma^2). \quad (6)$$

The combination of $\pm \mathcal{A}_\mu{}^i$ and Γ into a generally covariant derivative:

$$\mathcal{D}_\mu{}^\pm \mathcal{F}_{\lambda\nu} = \partial_\mu{}^\pm \mathcal{F}_{\lambda\nu} + [\pm \mathcal{A}_\mu, \pm \mathcal{F}_{\lambda\nu}] - \pm \mathcal{F}_{\lambda\sigma} \Gamma^\sigma{}_{\nu\mu} - \pm \mathcal{F}_{\sigma\nu} \Gamma^\sigma{}_{\lambda\mu} \quad (7)$$

leaves the $su(2)$ -projectors invariant:

$$\mathcal{D}_\mu({}^\pm Z_i)_{\lambda\nu} = \nabla_\mu({}^\pm Z_i)_{\lambda\nu} + \epsilon_{ijk}{}^\pm \mathcal{A}_\mu{}^j({}^\pm Z^k)_{\lambda\nu} = 0.$$

As a consequence covariant differentiation and $su(2)$ reduction commute:

$$\mathcal{D}_\mu{}^\pm \mathcal{F}_{\lambda\nu} = -({}^\pm Z_i)^\alpha{}_\beta \nabla_\mu R^\beta{}_{\alpha\lambda\nu}. \quad (8)$$

Furthermore, the gauge covariant divergence and the generally covariant divergence of the field strength are related in a simple way:

$$D_\mu{}^\pm \mathcal{F}^\mu{}_{\nu i} = \frac{1}{\sigma^2} \mathcal{D}_\mu{}^\pm \mathcal{F}^\mu{}_{\nu i}. \quad (9)$$

Equations (8,9) imply a simple relation between the divergences of \mathcal{F} and R :

$$D_\mu^\pm \mathcal{F}^\mu_{\nu i} = -\frac{1}{\sigma^2} (\pm Z_i)^\alpha_\beta \nabla^\mu R^\beta_{\alpha\mu\nu} \quad (10)$$

and this yields a translation of the Yang–Mills equations into a condition on the curvature of the conformally flat space. The vanishing of the Weyl tensor implies that the covariant divergence of the Riemannian in a conformally flat space is

$$\nabla^\mu R^\beta_{\alpha\mu\nu} = -\frac{1}{6} (\delta^\beta_\nu \partial_\alpha - \delta_{\alpha\nu} \partial^\beta) R$$

with the curvature scalar $R = R^\mu_\mu$. Therefore the gauge covariant divergence of the su(2) curvatures from the conformally flat space are simply proportional to the gradient of the curvature scalar:

$$D_\mu^\pm \mathcal{F}^\mu_{\nu i}(x) = \frac{1}{3\sigma^2(x)} (\pm Z_i)^\mu_\nu \partial_\mu R(x) \quad (11)$$

and this expresses Wilczek’s theorem [109]: su(2) reduction of the Riemannian connection in a conformally flat 4-space yields solutions to the Yang–Mills equations if and only if the curvature scalar is constant.

If the curvature scalar is expressed in terms of the conformal factor χ this theorem establishes a connection between Yang–Mills theory and χ^4 theory:

$$\partial_\mu \partial^\mu \chi + \frac{1}{6} R \chi^3 = 0. \quad (12)$$

However, the formulation in terms of the inverse conformal factor serves our purposes better:

$$\partial_\mu \partial^\mu \sigma^2 - \frac{3}{2\sigma^2} \partial_\mu \sigma^2 \cdot \partial^\mu \sigma^2 - \frac{1}{3} R = 0. \quad (13)$$

We can not solve this equation in a general fashion. However, it was observed in [32] that the stronger condition of local symmetry

$$\nabla_\lambda R^\mu_\nu = 0$$

allows for a general solution. In terms of $\sigma(x)$ the condition of local symmetry takes the following form:

$$\partial^\mu \partial_\nu \partial_\lambda \sigma^2 + \delta^\mu_\nu \partial_\lambda [\sigma \partial_\alpha \partial^\alpha \sigma - 2(\partial_\alpha \sigma)(\partial^\alpha \sigma)] - [\delta^\mu_\nu \partial_\lambda + \delta^\mu_\lambda \partial_\nu + \delta_{\lambda\nu} \partial^\mu] (\partial_\alpha \sigma)(\partial^\alpha \sigma) = 0. \quad (14)$$

Summation over μ and ν leads again to

$$\partial_\lambda R = 6\partial_\lambda [\sigma \partial_\alpha \partial^\alpha \sigma - 2(\partial_\alpha \sigma)(\partial^\alpha \sigma)] = 3\partial_\lambda [\partial_\alpha \partial^\alpha \sigma^2 - 6(\partial_\alpha \sigma)(\partial^\alpha \sigma)] = 0 \quad (15)$$

and this shows that equation (14) is equivalent to a set of equations consisting of (13) and

$$\partial_\mu \partial_\nu \partial_\lambda \sigma^2 - \frac{1}{6} (\delta_{\mu\nu} \partial_\lambda + \delta_{\mu\lambda} \partial_\nu + \delta_{\lambda\nu} \partial_\mu) \partial_\alpha \partial^\alpha \sigma^2 = 0. \quad (16)$$

Eq. (16) can be solved by standard methods, and the simplest method of solution assuming differentiability to fifth order proceeds as follows:

Contracting (16) with ∂^λ and with $\partial^\lambda \partial^\nu$ yields

$$\partial_\mu \partial_\nu \partial_\lambda \partial^\lambda \sigma^2 = \frac{1}{4} \delta_{\mu\nu} (\partial_\lambda \partial^\lambda)^2 \sigma^2$$

$$\partial_\mu (\partial_\lambda \partial^\lambda)^2 \sigma^2 = 0$$

implying

$$\partial_\mu \partial_\nu \partial_\rho \partial^\rho \sigma^2 = 48\lambda^2 \delta_{\mu\nu}$$

where in writing the arbitrary integration constant as a square we already took into account that our solution in the end should correspond to a definite function. The previous equation is readily integrated to

$$\partial_\nu \partial_\lambda \partial^\lambda \sigma^2 = 48\lambda^2 x_\nu + 12c_\nu$$

whence (16) reduces to

$$\partial_\mu \partial_\nu \partial_\rho \sigma^2 = \delta_{\mu\nu} (8\lambda^2 x_\rho + 2c_\rho) + \delta_{\mu\rho} (8\lambda^2 x_\nu + 2c_\nu) + \delta_{\rho\nu} (8\lambda^2 x_\mu + 2c_\mu).$$

This is readily integrated again and the general solution contains 20 parameters:

$$\sigma^2(x) = \lambda^2 r^4 + c_\mu x^\mu r^2 + \frac{1}{2} a_{\mu\nu} x^\mu x^\nu + b_\mu x^\mu + \zeta^2. \quad (17)$$

We may identify the 4-vector c and the off-diagonal components of a as Poincaré degrees of freedom, and gauge them away through an appropriate Poincaré transformation. We then write $a_{\mu\nu} = \Lambda_\mu \delta_{\mu\nu}$ and use the abbreviation $A \equiv \Sigma_\mu \Lambda_\mu - \frac{R}{3}$. In this gauge equation (13) translates into the following set of algebraic constraints on the coefficients in (17):

$$\begin{aligned} A\lambda^2 &= 0, \\ 2A\zeta^2 &= 3b_\mu b^\mu, \\ \lambda^2 b_\mu &= 0, \\ (3\Lambda_\mu - A)b_\mu &= 0, \\ (3\Lambda_\mu - A)\Lambda_\mu &= 48\lambda^2 \zeta^2. \end{aligned}$$

In case $\lambda^2 = 0$ there exist only singular solutions with either a pointlike singularity (corresponding to the meron solution of De Alfaro, Fubini and Furlan [29]), singular lines, planes or 3-spaces. In case $\lambda^2 > 0$ there exist singular solutions with either two pointlike singularities (2-meron solution), a singular circle, a singular 2-sphere, or a singular 3-sphere. However, there exists one regular solution given by

$$\sigma(x) = \lambda(r^2 + \varrho^2). \quad (18)$$

The corresponding metric describes a 4-sphere of radius $r_4 = \frac{1}{2\lambda\varrho}$ centered at a point $(\varrho - r_4)\vec{e}_5$ in \mathbb{R}^5 in stereographic coordinates. The BPST instanton $^+\mathcal{A}$ and anti-instanton $^-\mathcal{A}$ are

$$^{\pm}\mathcal{A}_{\mu i} = \frac{2}{r^2 + \varrho^2}(\pm\delta^0_{\mu}x_i \mp \delta_{i\mu}x^0 + \epsilon_{ijk}\delta^j_{\mu}x^k) \quad (19)$$

and they satisfy

$$\lim_{r \rightarrow \infty} ^+\mathcal{A}_{\mu i}(x) \frac{\sigma^i}{2i} = U^{-1}(x) \partial_{\mu} U(x), \quad \lim_{r \rightarrow \infty} ^-\mathcal{A}_{\mu i}(x) \frac{\sigma^i}{2i} = U(x) \partial_{\mu} U^{-1}(x),$$

with

$$U(x) = \frac{1}{r}(x^0 + x^i \sigma_i).$$

Writing the angle to \vec{e}_0 , $\cos \vartheta = \frac{x^0}{r}$, it is apparent that

$$U^{\pm n}(x) = \cos(n\vartheta) \pm i \frac{x^i \sigma_i}{\sqrt{x_j x^j}} \sin(n\vartheta)$$

describe mappings of winding numbers $\pm n$ from S^3 to $\text{SU}(2)$.

The corresponding field strengths are

$$^{\pm}E_j^{i} = \pm \frac{4\varrho^2}{q(r^2 + \varrho^2)^2} \delta_j^{i}, \quad ^{\pm}B_j^{i} = -\frac{4\varrho^2}{q(r^2 + \varrho^2)^2} \delta_j^{i}, \quad (20)$$

and these solutions are apparently (anti-)selfdual under the euclidean duality transformation $E \rightarrow -B$, $B \rightarrow -E$. In the present construction the duality property arises as a consequence of the diagonal structure of the field strengths with respect to space-time and $\text{su}(2)$ indices. This ensures that the duality of $^{\pm}\mathcal{F}^{\alpha}_{\beta\mu\nu} \equiv ^{\pm}\mathcal{F}^i_{\mu\nu}(\pm Z_i)^{\alpha}_{\beta}$ in the internal indices α, β carries over to the space-time indices μ, ν .

The energy density is

$$\mathcal{L} = \frac{48\varrho^4}{q^2(r^2 + \varrho^2)^4}$$

yielding an action

$$S = \frac{8\pi^2}{q^2}.$$

These solutions and their generalizations to higher winding numbers n appear also in general $\text{SU}(N_c)$ gauge theory through the various embeddings of $\text{SU}(2)$ subgroups.

It is apparent then that instantons should contribute to the effective dilaton potential since the dilaton couples directly to F^2 and a large dilaton would imply large instanton energy.

The singular solutions with $\lambda^2 \neq 0$ also fall off like r^{-4} at large distances and therefore really correspond to solutions of the vacuum Yang-Mills equations, albeit with infinite action.

3 The Kaluza–Klein paradigm

Kaluza–Klein theory asserts that part of the scalar and vector fields and the metric in a theory in d dimensions can be identified as components of a higher–dimensional metric, and that the appropriate Lagrangian in d dimensions can be inferred from a higher–dimensional theory containing additional compact dimensions. One might distinguish a strong and a weak version of Kaluza–Klein theory: The strong version would suppose that the d –dimensional inverse metric tensor is directly embedded in the corresponding higher–dimensional inverse metric without rescaling. This would require a direct coupling of scalars to the Einstein–Hilbert term in the low–dimensional theory. The weak version would permit field dependent reparametrizations between the actual d –dimensional metric and the metric inherited from higher dimensions.

In the more recent history of physics, interest in the dilaton revived in the realm of Kaluza–Klein supergravity. If the field content of space–time is assumed to arise from embeddings in a $(4 + d_i)$ –dimensional manifold of Minkowski signature with a compact factor, the d_i –dimensional internal manifold will have variable volume from the four–dimensional point of view, and this variable volume will be encoded in a local field $\phi(x)$, the dilaton. However, my expectation is that a low–dimensional Kaluza–Klein dilaton tells us only about local variations of the volume of the internal manifold, its four–dimensional dynamics can not fix the mean value of that volume. Explaining and fixing the smallness of internal dimensions belongs to the realm of Kaluza–Klein cosmology or string theory in the $(4 + d_i)$ –dimensional framework. Nevertheless, the low–dimensional field theory must be self-consistent and tell us why variations of the internal dimensions are small or invisible, and this problem concerns the issue of the effective potential of Kaluza–Klein type dilatons in four dimensions.

While Kaluza–Klein theory is not a suitable framework for a unified theory of fundamental interactions, it still provides an indispensable tool in field theoretical investigations of string theory, where compact manifolds with or without boundaries arise both in low energy effective field theories and in the net of extended objects with p spatial dimensions, so called p –branes. The dynamics of nets of various p –branes in $D \leq 11$ dimensions is currently under close investigation as a paradigm for non-perturbative effects in string theory. Far away from the intersections the excitations of the p –branes can be described in terms of $p + 1$ –dimensional field theories on the branes, and compactness arises for those dimensions which describe the extension of a given p –brane between other extended objects.

Quantum field theory on manifolds with compact directions is an old subject of theoretical physics, with an almost canonical structure in the bosonic sector and some matters of taste in the more complicated fermionic sector. I will give an account on the compactification of one dimension in my favorite conventions, and the cogniscenti may find it amusing (or bothersome) to compare to their favorite frameworks. We will go from D to $D - 1$ dimensions through compactification on a circle, and assume $D \geq 4$ in the sequel. As long as only zero modes of internal derivatives are taken into account repeated application of the following formulas can be used to parametrize any Kaluza–Klein theory ending up in $d \geq 3$

dimensions. However, besides compactifications from 5 to 4 and from 4 to 3 dimensions we will need only a few selected results for compactifications from $D \geq 6$ to 4 dimensions.

In addition to serving as a generator for torus compactifications there are further reasons to go from D to $D - 1$ dimensions: My first motivation for this arose from recent developments in string theory: Motivated by the successful applications of duality symmetries in the investigation of low energy effective actions of supersymmetric gauge theories, Witten observed that theories with Bogomol'nyi saturated solitons should have dual descriptions in terms of Kaluza–Klein theories, in order to explain the equidistant mass spectrum of particles in the dual theory [113, 114]. This fits very well with proposals by Witten and by Hořava and Witten, saying that large dilaton limits of type *IIA* and heterotic $E_8 \times E_8$ superstring theory are described by large radius compactifications of an eleven-dimensional theory on $M_{10} \times S^1$ or $M_{10} \times S^1/\mathbb{Z}_2$, which has been called M-theory [113, 60]. While this initiated the current activity to define or identify M-theory as a matrix theory or as a theory treating extended objects of various dimensions as (almost) equally fundamental degrees of freedom³, it also motivated speculations about relations between supersymmetric theories in three dimensions and non-supersymmetric theories in four dimensions, including in particular a proposal to solve the cosmological constant problem through supersymmetry in three dimensions [115]. While there is no complete picture yet concerning the impact of different sectors of the moduli space of M-theory and string theory on low energy physics, it implies existence of parametrizations of low energy effective field theory which implement a dilaton either in three or in four dimensions⁴. It has been stressed by Banks and Dine that four-dimensional compactifications of M-theory should imply a five-dimensional threshold two orders of magnitude below the GUT scale, i.e. around 10^{14} GeV [8], implying that quantum gravity effects may become strong before unification is achieved in a field theoretic framework. In a recent update on Kaplunovsky's work on large volume string compactifications Caceres et al. also discuss a five-dimensional threshold in heterotic string theory [22].

A Kaluza–Klein decomposition of the metric which preserves the Einstein-frame is

$$G_{MN} = \Phi^{-\frac{1}{D-2}} \begin{pmatrix} g_{\mu\nu} + \Phi a_\mu a_\nu & \Phi a_\mu \\ \Phi a_\nu & \Phi \end{pmatrix}. \quad (21)$$

A priori this is a mere reparametrization of the metric comparable to an ADM decomposition. It becomes an Ansatz if we assume that the fields do not depend on x^{D-1} or their dependence on x^{D-1} is negligible, due to translational invariance or compactness along

³My impression is that strings still play a more fundamental role than other extended objects, since their spectra determine the brane-scan.

⁴Even in the Einstein frame a radially coupled dilaton in a field theory on $M_d \times X \times S^1$ looks like a radially coupled dilaton from an Einstein frame on $M_d \times S^1$ only in a very particular class of parametrizations of the Kaluza–Klein Ansatz. Generically, the large dilaton limit will look like a scalar–vector–tensor theory of gravity on $M_d \times S^1$.

x^{D-1} . Eq. (21) then yields for the zero modes of ∂_{D-1} and up to surface terms⁵

$$\sqrt{-G} G^{MN} R_{MN} = \sqrt{-g} [g^{\mu\nu} R_{\mu\nu} - \frac{D-3}{4(D-2)} g^{\mu\nu} \partial_\mu \ln \Phi \cdot \partial_\nu \ln \Phi - \frac{\Phi}{4} f_{\mu\nu} f^{\mu\nu}], \quad (22)$$

while reduction of the Yang–Mills field $A_M{}^k \rightarrow A_\mu{}^k, A^k$ yields

$$-\frac{1}{4} \sqrt{-G} F_{MN}{}^k F^{MN}{}_k = -\frac{1}{4} \sqrt{-g} \Phi^{\frac{1}{D-2}} [B_{\mu\nu}{}^k B^{\mu\nu}{}_k + \frac{2}{\Phi} g^{\mu\nu} D_\mu A^k \cdot D_\nu A_k], \quad (23)$$

$$B_{\mu\nu} = F_{\mu\nu} + a_\mu D_\nu A - a_\nu D_\mu A.$$

We follow the usual terminology to denote the mechanism leading from Einstein gravity in D dimensions to Einstein–Yang–Mills theory in $D-1$ dimensions as compactification on a circle. This has to be qualified in three directions: First one should keep in mind that Einstein gravity actually induces a scalar–vector–tensor type theory of gravity on submanifolds, which is not strictly a scalar–vector–tensor theory since the scalar and vector degrees of freedom also couple directly to matter. In the scalar–tensor sector the induced theory would resemble a Jordan–Brans–Dicke theory as a consequence of the fact that the metric on the submanifold induced from the metric G is

$$\tilde{g}_{\mu\nu} = \Phi^{-\frac{1}{D-2}} (g_{\mu\nu} + \Phi a_\mu a_\nu).$$

If we insist that Kaluza–Klein theory yields Einstein gravity in lower dimensions we suppose that the physical metric on the submanifold is not the metric inherited from the embedding space. Stated differently, the geodesics traced out by test particles are not the geodesics of the embedded submanifold.

The second remark concerns the naive picture of a product manifold $\mathcal{M} \times \mathcal{X}$, with \mathcal{X} compact. While this is a particular possibility considered by Kaluza–Klein theory, it is not the most general setting. Generically, the compact manifold \mathcal{X} acts as a fiber in a total space which projects to \mathcal{M} . Indeed, in the case of a product manifold the vector fields a could be safely neglected if the Riemannian structure respects the product structure: The connection coefficients $\Gamma^{D-1}{}_{\mu\nu}$ and $\Gamma^\mu{}_{D-1\nu}$ are gauge equivalent to zero for arbitrary Φ if and only if a is a gradient. This is equivalent to the requirement that vectors tangent and normal to the internal dimensions are mapped to tangent and normal vectors under parallel translation.

The third remark also concerns the graviphotons a : To lowest order in scalar contributions to the metric these vector fields contribute Maxwell terms to the low-dimensional Lagrangian. However, fermions will be neutral with respect to these gauge fields. Instead, the graviphoton mixes with gauge fields inherited from the higher-dimensional theory in such a way that the low-dimensional gauge field becomes neutral under diffeomorphisms

⁵Whenever equations are written down for terms appearing in a Lagrangian, we will consider expressions equivalent if they differ by a divergence and neglect the divergences in the sequel.

Compactification to the Einstein frame becomes singular for $D=3$, and other Ansätze have to be employed in this case, see e.g. [17, 71, 72].

normal to the submanifold. This can be seen explicitly in the reduction of fermion contributions to the Lagrangian carried out below.

A detailed discussion of the reduction of fermion terms requires a distinction between even and odd values of D . My favorite choice for embeddings and reductions of γ -matrices is based on Weyl bases in even dimensions and Dirac bases in odd dimensions. General discussions of properties of spinors in arbitrary dimensions can be found in [73, 107].

If D is **even** and γ_μ is a basis of Dirac matrices in $D - 1$ dimensions, a Weyl basis of Dirac matrices in D dimensions is given by

$$\begin{aligned}\Gamma_0 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \Gamma_j &= \begin{pmatrix} 0 & -\gamma_0 \gamma_j \\ \gamma_0 \gamma_j & 0 \end{pmatrix} \\ \Gamma_{D-1} &= \begin{pmatrix} 0 & -\gamma_0 \\ \gamma_0 & 0 \end{pmatrix}.\end{aligned}$$

The corresponding analog of γ_5 is

$$\Gamma_{D+1} = i^{\frac{D}{2}+1} \Gamma_0 \cdot \Gamma_1 \dots \Gamma_{D-1}.$$

If we start with a $(1 + 0)$ -dimensional γ -matrix $\gamma_0 = -1$, and with the conventions for γ -matrices in odd dimensions described below, Γ_{D+1} takes the form

$$\Gamma_{D+1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

This embedding of γ -matrices provides a direct mapping between the spinor representation in D dimensions and both inequivalent representations in $D - 1$ dimensions, and the appropriate Ansatz for the dimensional reduction of the spinor field is

$$\Psi = \Phi^{\frac{1}{4(D-2)}} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}.$$

In a gauge $E^\mu_{D-1} = 0$ for the D -bein the dimensionally reduced action reads

$$\begin{aligned}& \sqrt{-G} \bar{\Psi} [E^M{}_A \Gamma^A (i\partial_M + i\Omega_M + qA_M) - M] \Psi = \\&= \sqrt{-g} [\bar{\psi}_+ e^\mu{}_a \gamma_+^a (i\partial_\mu + i\omega_{+\mu} + qV_\mu) \psi_+ + \bar{\psi}_- e^\mu{}_a \gamma_-^a (i\partial_\mu + i\omega_{-\mu} + qV_\mu) \psi_-] \\&+ q\sqrt{-g} \Phi^{-\frac{1}{2}} (\bar{\psi}_+ A \psi_+ + \bar{\psi}_- A \psi_-) + M\sqrt{-g} \Phi^{-\frac{1}{2(D-2)}} (\psi_+^\dagger \psi_- + \psi_-^\dagger \psi_+) \\&- \frac{i}{8} \sqrt{-g} \Phi^{\frac{1}{2}} f_{ab} (\bar{\psi}_+ \gamma_+^{ab} \psi_+ + \bar{\psi}_- \gamma_-^{ab} \psi_-)\end{aligned} \tag{24}$$

with $\gamma_\pm^0 = \pm \gamma^0$, $\gamma_\pm^j = \gamma^j$, while

$$\Omega_\mu = -\frac{1}{4} \Gamma^A \Gamma^B \Omega_{AB\mu}$$

and

$$\omega_{\pm\mu} = -\frac{1}{4}\gamma_{\pm}^a\gamma_{\pm}^b\omega_{ab\mu}$$

denote the canonical spin connections for the metrics G and g , respectively. The vector field V_{μ} appearing as a gauge potential in $D-1$ dimensions is

$$V_{\mu} = A_{\mu} - a_{\mu}A,$$

$$V_{\mu\nu} = B_{\mu\nu} - Af_{\mu\nu}.$$

Note that $D_{\mu}A_j = \partial_{\mu}A_j - qA_{\mu}^i f_{ij}^k A_k = \partial_{\mu}A_j - qV_{\mu}^i f_{ij}^k A_k$.

If D is **odd** and γ_{μ} is a basis of Dirac matrices in $D-1$ dimensions, a basis of Dirac matrices in D dimensions is given by

$$\Gamma_0 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Gamma_j = \gamma_j, \quad 1 \leq j \leq D-2$$

$$\Gamma_{D-1} = -i\gamma_0.$$

We have

$$i^{\frac{D+1}{2}+1}\Gamma_0 \cdot \Gamma_1 \dots \Gamma_{D-1} = 1$$

and a second inequivalent basis is given by $-\Gamma_0, \Gamma_J, 1 \leq J \leq D-1$.

Contrary to the embedding for even D described above, this embedding does not provide a direct mapping between spinor representations of the Lorentz group in D and $D-1$ dimensions, and we have to compensate for that through an extra factor \mathcal{X} in the Kaluza-Klein Ansatz for spinors:

$$\Psi = \Phi^{\frac{1}{4(D-2)}}\mathcal{X} \cdot \psi,$$

$$\mathcal{X}^+ \cdot \Gamma_0 \cdot \mathcal{X} = \gamma_0,$$

$$\mathcal{X}^+ \cdot \Gamma_j \cdot \mathcal{X} = \gamma_j, \quad 1 \leq j \leq D-2.$$

In the bases of γ -matrices employed here, \mathcal{X} is realized explicitly as

$$\mathcal{X} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}.$$

Employing again a gauge $E^{\mu}_{D-1} = 0$ for the D -bein the reduced action reads

$$\begin{aligned} & \sqrt{-G}\bar{\Psi}[E^M{}_A\Gamma^A(i\partial_M + i\Omega_M + qA_M) - M]\Psi = \\ & = \sqrt{-g}[\bar{\psi}e^{\mu}{}_a\gamma^a(i\partial_{\mu} + i\omega_{\mu} + qV_{\mu})\psi] - iq\sqrt{-g}\Phi^{-\frac{1}{2}}\bar{\psi}\gamma_D A\psi \\ & \quad - M\sqrt{-g}\Phi^{-\frac{1}{2(D-2)}}\bar{\psi}\psi - \frac{1}{8}\sqrt{-g}\Phi^{\frac{1}{2}}f_{ab}\bar{\psi}\gamma^{ab}\gamma_D\psi \end{aligned} \tag{25}$$

with Ω and ω denoting the canonical spin connections for the metrics G and g , respectively, and again $V_\mu = A_\mu - a_\mu A$.

Since time reversal in odd dimensions mixes the two equivalence class of representations of the corresponding Clifford algebra, one has to add a second spinor Ψ_- related to $\Gamma_-^0 = -\Gamma^0$, $\Gamma_-^J = \Gamma^J$ and a corresponding low-dimensional spinor ψ_- through

$$\Psi_- = \Phi^{\frac{1}{4(D-2)}} \mathcal{X}_- \cdot \psi_-$$

with

$$\mathcal{X}_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$

This yields then the same low-dimensional action with the sign of the parity violating terms inverted. To end up with an irreducible representation of the Lorentz group in $D-1$ dimensions we impose the constraint $\psi_- = \psi$, whence our $(D-1)$ -dimensional action is

$$\mathcal{L} = \sqrt{-g} [\bar{\psi} e^\mu_a \gamma^a (i\partial_\mu + i\omega_\mu + qV_\mu) \psi] - M \sqrt{-g} \Phi^{-\frac{1}{2(D-2)}} \bar{\psi} \psi.$$

Both for even and odd values of D we have not encountered a photon-like coupling of the graviphoton to low-dimensional fermions. This can be understood very easily: The only place where a photon-type coupling of the graviphoton could arise on zero modes of ∂_{D-1} is in the reduction of the spin-connection, through terms containing

$$E^{D-1}_a = -\Phi^{\frac{1}{2(D-2)}} a_\mu e^\mu_a.$$

However, all these terms contain derivatives on Φ , e^μ_a or a_μ and can not create a $U(1)$ gauge coupling. On the zero modes components of the metric tensor in higher dimensions modify gauge fields upon compactification, but they do not create new gauge couplings to fermions beyond those couplings already present in higher dimensions. Of course, a possible way to avoid this negative verdict on Kaluza-Klein generated gauge fields relies on eigenspinors of the internal Dirac operator $E^{D-1}_a \gamma^a \partial_{D-1}$, but this will play no role in the sequel.

In concluding this section I would like to add a remark on the string frame in four dimensions: String theory in the string frame supposes that the dilaton couplings in four dimensions look like Kaluza-Klein couplings with the four-dimensional metric directly induced from an embedding space, i.e. it is based on the strong rather than the weak version of Kaluza-Klein theory. Neglecting the gauge fields and denoting by $\Phi_{\mathcal{BD}}^2$ the determinant of the internal metric, compactification from D dimensions to the Einstein frame and a string-like frame in four dimensions would proceed via

$$G_{MN} = \Phi_{\mathcal{BD}}^{-1} \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & \Phi_{\mathcal{BD}}^{\frac{D-2}{D-4}} h_{mn} \end{pmatrix} = \begin{pmatrix} \tilde{g}_{\mu\nu} & 0 \\ 0 & \Phi_{\mathcal{BD}}^{\frac{2}{D-4}} h_{mn} \end{pmatrix},$$

respectively, and yield

$$\begin{aligned} \sqrt{-G} G^{MN} R_{MN} &= \sqrt{-g} g^{\mu\nu} (R_{\mu\nu} - \frac{D-2}{2(D-4)} \partial_\mu \ln \Phi_{\mathcal{BD}} \cdot \partial_\nu \ln \Phi_{\mathcal{BD}}) \\ &= \sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \Phi_{\mathcal{BD}} (\tilde{R}_{\mu\nu} + \frac{D-5}{D-4} \partial_\mu \ln \Phi_{\mathcal{BD}} \cdot \partial_\nu \ln \Phi_{\mathcal{BD}}). \end{aligned} \quad (26)$$

Therefore, in strong Kaluza–Klein theory gravity would be described by a Brans–Dicke type theory with a Brans–Dicke parameter

$$\omega_{\mathcal{BD}} = -\frac{D-5}{D-4}.$$

This is not yet gravity in the string frame, since both $\Phi_{\mathcal{BD}}$ and the string dilaton ϕ_S couple to the Yang–Mills terms, and the metric $\tilde{g}_{\mu\nu}$ has to be Weyl rescaled by $\exp(\sqrt{\frac{\kappa}{2}}\phi_S)$ to ensure equal coupling both to R and to F^2 . The resulting theory in the gravitational sector is a Brans–Dicke theory (1) with

$$\omega_{\mathcal{BD}} = -1,$$

and initially $\Lambda(\Phi_{\mathcal{BD}}) = 0$. However, Brans–Dicke theory is constrained by the fact that solar system tests of gravity restrict Brans–Dicke parameters for massless Brans–Dicke scalars to $\omega_{\mathcal{BD}} > 500$ [111]. This leaves two possibilities for the string frame: We either should identify a mechanism to generate a large mass for $\Phi_{\mathcal{BD}}$ if low energy gravity in string theory is described by a scalar–tensor type theory in the string frame, or higher loop effects in string theory modify the Brans–Dicke coupling function from $R\Phi_{\mathcal{BD}}$ to a convex function $C(\Phi_{\mathcal{BD}})R$. In the second case the cosmological attractor mechanism of Damour and Nordtvedt would apply [28], and cosmological evolution would restore effective Einstein gravity in the string frame.

Our main interest in the present work is in light dilatons, and therefore we rely on the conservative assumption that low energy gravity is described by Einstein gravity.

Compactification of heterotic string theory in the Einstein frame shows that the four–dimensional dilaton ϕ coupling to gauge fields arises as a linear combination of the string dilaton ϕ_S already present in the ten–dimensional field theory limit and a Kaluza–Klein dilaton ϕ_K arising from the compactification to four dimensions [112]. If both ϕ_S and ϕ_K are normalized to have standard kinetic terms in four dimensions the dilaton is dominated by the Kaluza–Klein component with a mixing angle $\theta_\phi = -\frac{\pi}{6}$:

$$\phi = \frac{1}{2}(\sqrt{3}\phi_K - \phi_S),$$

and its decay constant is

$$f_\phi = \frac{m_{Pl}}{\sqrt{2}}. \tag{27}$$

This relies on a parametrization for the string dilaton such that strong gauge coupling in ten dimensions corresponds to strongly coupled string theory and is based on the fact that the ten–dimensional field theory limit of heterotic string theory consists of $N = 1$ supergravity coupled to $N = 1$ supersymmetric gauge theory [56, 58, 53]. This theory contains pieces derived from eleven–dimensional supergravity, but the string dilaton couples stronger than a Kaluza–Klein dilaton from eleven dimensions, and for this reason the dilaton

decay constant in four dimensions realizes a lower bound for Kaluza–Klein decay constants arising through compactifications from D dimensions:

$$f_{\phi(KK)} = \frac{m_{Pl}}{\sqrt{2}} \sqrt{\frac{D-2}{D-4}} > f_{\phi}.$$

The alert reader may wonder why neither in f_{ϕ} nor in any pure Kaluza–Klein decay constant any compactification scales show up. This is due to the fact that internal volumes rescale higher–dimensional gravitational constants to the four–dimensional constant κ , and in normalizing dilatons to standard kinetic terms only a rescaling with m_{Pl} is involved. Therefore the decay constants only depend on the Planck mass.

A dilaton coupling scale (27) of the order of the Planck mass implies invisibility of the dilaton from the particle physics point of view: We can readily calculate the integrated tree level cross section for creation of a dilaton pair through head–on collision of two gauge bosons with Mandelstam parameter s

$$\sigma = \frac{s}{64\pi f_{\phi}^4},$$

and this tells us that even for Planck scale collisions the cross section would be tiny $\sigma \simeq \frac{1}{4\pi} l_{Pl}^2$.

4 The dilaton in three dimensions

Interest in a three-dimensional dilaton arose from Witten's observation that theories with Bogomol'nyi saturated solitons may be related to theories in higher dimensions in a Kaluza-Klein type framework [113, 114]. This idea can be motivated from classical duality considerations which generically imply a trading between solitons and particles in dual theories. Given the soliton-particle correspondence and the infinite tower of equidistant solitonic excitations it seems very natural to relate solitons with an equidistant mass spectrum to compactified theories in higher dimensions. In this framework the particular case of dualities between supersymmetric theories in 2+1 dimensions and non-supersymmetric theories in 3+1 dimensions deserves special attention, since the four-dimensional theory might inherit the vanishing of the cosmological constant from the corresponding three-dimensional theory⁶ [114, 115]. A discussion of the supersymmetric abelian Higgs model in 2+1 dimensions coupled to supergravity confirmed this picture by showing that the soliton spectrum in this theory is not supersymmetric [11]. It may also be worth-while to point out that due to the topological nature of gravity in three dimensions one would not need a fully fledged supergravity multiplet to get rid of the cosmological constant in this scenario. In particular, we would not need a gravitino, which would be hard to accomodate in the four-dimensional theory.

However, soon after Witten's proposal worries arose that the static potential in 2+1 dimensions would imply a logarithmic divergence of the dilaton for any static source. In spite of that it turned out that the infrared singularity of the propagator actually suppresses fluctuations of the dilaton in three dimensions. The suppression of fluctuations works because fermions and adjoint scalars provide sources for the dilaton which differ in sign from the dilaton sources provided by the gauge fields arising from the four-dimensional metric and four-dimensional gluons. Finite energy or independence of the theory from an infrared regulator then implies that any local dilaton source has to be compensated by another dilaton source somewhere else, whence the dilaton vanishes asymptotically and the radius of the internal dimension would approach a value to be fixed by string theory.

The idea to get rid of a dilaton through duality symmetries between theories in different dimensions is very speculative, but the mechanism outlined here [33] provides an example for non-perturbative stabilization of a dilaton in a low energy effective theory different from the mechanisms outlined in section 6.

In order to make the observations outlined above quantitative, I will take a four-dimensional point of view and discuss the action of the three-dimensional dilaton arising through a Kaluza-Klein parametrization of Einstein-Yang-Mills theory in four dimensions.

We relate the dilaton ϕ to the metric coefficient Φ via

$$\Phi = \exp(\sqrt{8\kappa}\phi),$$

⁶Supersymmetry as a solution to the cosmological constant problem has been discussed in [119]. A very useful review and critical discussion of several attempts to solve the problem can be found in [104].

where the three-dimensional gravitational constant κ is the four-dimensional gravitational constant divided by the circumference of the compact dimension. After appropriate rescalings of the other fields and coupling constants, we infer the following action in three dimensions from the results of the previous section:

$$\begin{aligned}
\frac{1}{\sqrt{-g}}\mathcal{L} = & \frac{1}{2\kappa}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi \cdot \partial_\nu\phi - \frac{1}{4}\exp(\sqrt{8\kappa}\phi)f_{\mu\nu}f^{\mu\nu} \\
& - \frac{1}{4}\exp(\sqrt{2\kappa}\phi)(V_{\mu\nu}{}^k V^{\mu\nu}{}_k + 2\sqrt{2\kappa}V_{\mu\nu}{}^k A_k f^{\mu\nu} + 2\kappa A_k A^k f_{\mu\nu} f^{\mu\nu}) \\
& + \bar{\psi}_+ e^\mu{}_a \gamma_+^a (i\partial_\mu + i\omega_{+\mu} + qV_\mu)\psi_+ + \bar{\psi}_- e^\mu{}_a \gamma_-^a (i\partial_\mu + i\omega_{-\mu} + qV_\mu)\psi_- \\
& + q\exp(-\sqrt{2\kappa}\phi)(\bar{\psi}_+ A\psi_+ + \bar{\psi}_- A\psi_-) + M\exp\left(-\sqrt{\frac{\kappa}{2}}\phi\right)(\psi_+^+ \psi_- + \psi_-^+ \psi_+) \\
& - \frac{1}{2}\exp(-\sqrt{2\kappa}\phi)g^{\mu\nu}D_\mu A_k D_\nu A^k - \frac{i}{8}\sqrt{2\kappa}\exp(\sqrt{2\kappa}\phi)f_{ab}(\bar{\psi}_+ \gamma_+^{ab}\psi_+ + \bar{\psi}_- \gamma_-^{ab}\psi_-).
\end{aligned}$$

In order to evaluate the effect of the infrared divergence of the electrostatic potential in 2+1 dimensions, we consider the energy of a static configuration with the fermions in stationary orbits, and in gauge $A_0 = a_0 = 0$ (complying with $E^\mu{}_3 = 0$, since we employ diffeomorphisms which are constant along the normal direction):

$$\begin{aligned}
\frac{1}{\sqrt{-g}}\mathcal{H} = & \frac{1}{2}g^{ij}\partial_i\phi \cdot \partial_j\phi + \frac{1}{4}\exp(\sqrt{2\kappa}\phi)(V_{ij}{}^k + \sqrt{2\kappa}A^k f_{ij})(V^{ij}{}_k + \sqrt{2\kappa}A_k f^{ij}) \\
& + \frac{1}{4}\exp(\sqrt{8\kappa}\phi)f_{ij}f^{ij} - \bar{\psi}_+ e^j{}_a \gamma_+^a (i\partial_j + i\omega_{+j} + qV_j)\psi_+ - \bar{\psi}_- e^j{}_a \gamma_-^a (i\partial_j + i\omega_{-j} + qV_j)\psi_- \\
& - q\exp(-\sqrt{2\kappa}\phi)(\bar{\psi}_+ A\psi_+ + \bar{\psi}_- A\psi_-) - M\exp\left(-\sqrt{\frac{\kappa}{2}}\phi\right)(\psi_+^+ \psi_- + \psi_-^+ \psi_+) \\
& + \frac{1}{2}\exp(-\sqrt{2\kappa}\phi)g^{ij}D_i A_k D_j A^k + \frac{i}{8}\sqrt{2\kappa}\exp(\sqrt{2\kappa}\phi)f_{ab}(\bar{\psi}_+ \gamma_+^{ab}\psi_+ + \bar{\psi}_- \gamma_-^{ab}\psi_-),
\end{aligned}$$

which tells us that the graviphoton and the Yang–Mills fields yield positive sources for the dilaton, while the kinetic term of A provides a negative contribution. There is some ambiguity with regard to the contribution due to the fermions. However, on-shell the fermion contribution adds up to a positive term. Therefore, generically the fermions will contribute negative sources to the dilaton.

This appearance of positive and negative source terms for the dilaton apparently works for any value of D . In ten-dimensional low energy effective actions of superstring theory involving only $N = 1$ supergravity the dilaton couples only with one sign since eleven-dimensional supergravity does not contain elementary fermions and Yang–Mills fields, and the residual 3-form potential and graviphoton are set to zero.

However, the case $D = 4$ is peculiar due to the logarithmic IR divergence of the electrostatic potential in 2+1 dimensions. In a linear approximation the dilaton $\ln\Phi$ behaves

like a massless scalar field coupled to external sources, whence finiteness of energy requires a vanishing dilaton charge:

$$\begin{aligned} \int d^2\mathbf{x} \varrho(\mathbf{x}) = \int d^2\mathbf{x} \Big(& \sqrt{\frac{\kappa}{2}} f_{ij} f^{ij} + \sqrt{\frac{\kappa}{8}} (V + \sqrt{2\kappa} A f)_{ij}{}^k (V + \sqrt{2\kappa} A f)^{ij}{}_k \\ & - \sqrt{\frac{\kappa}{2}} g^{ij} D_i A^k \cdot D_j A_k + \sqrt{2\kappa} q (\bar{\psi}_+ A \psi_+ + \bar{\psi}_- A \psi_-) \\ & + \sqrt{\frac{\kappa}{2}} M (\psi_+^\dagger \psi_- + \psi_-^\dagger \psi_+) + \frac{i}{4} \kappa f_{ab} (\bar{\psi}_+ \gamma_+^{ab} \psi_+ + \bar{\psi}_- \gamma_-^{ab} \psi_-) \Big) = 0. \end{aligned} \quad (28)$$

This property is similar to charge neutrality of the Coulomb gas in two dimensions and can be derived from conformal invariance of the partition function or independence of the length scale λ entering the definition of the electrostatic potential:

$$\Phi(\mathbf{x}) = \exp \left(\frac{\sqrt{2\kappa}}{\pi} \int d^2\mathbf{x}' \varrho(\mathbf{x}') \ln \frac{|\mathbf{x} - \mathbf{x}'|}{\lambda} \right).$$

Eq. (28) is equivalent to absence of a logarithmic singularity of the dilaton.

If one feels uncomfortable about the use of a linear approximation in this argument one may alternatively rely on independence of the perturbation theory on the scale λ . λ appears as an infrared cutoff if the static result is inferred from the retarded potential

$$G(t - t', \mathbf{x} - \mathbf{x}') = \frac{\Theta((t - t') - |\mathbf{x} - \mathbf{x}'|)}{2\pi \sqrt{(t - t')^2 - |\mathbf{x} - \mathbf{x}'|^2}}.$$

The self-trapping mechanism encoded in (28) is superficially stable with respect to quantum effects, since calculation of the 1-loop effective potential in dimensional regularization yields no perturbatively generated effective potential. This is explained for four dimensions in appendix B. I would also like to point out that the physics behind (28) is more transparent than in the case of the Coulomb gas, since particles and antiparticles contribute in the same way to the dilaton: If a gauge boson excites a dilaton field the divergence of the resulting energy density implies pair production of adjoint scalars and fermions to restore an asymptotically vanishing dilaton. Clearly, M suppresses the production of fermions relative to adjoint scalars. Stated in another way: The adjoint scalar and light fermions screen the dilaton charge of the gauge bosons.

Compactification to three dimensions and subsequent decompactification due to BPS solitons is an interesting, but speculative proposal for the low energy sector of string theory, and we will concentrate on the four-dimensional dilaton in the sequel.

5 Generalized Coulomb potentials in gauge theory with a dilaton

As pointed out in the previous sections we expect dilatonic degrees of freedom in four-dimensional gauge theories if physics at very high energies involves decompactification of internal dimensions or string theory. To acquire a better understanding of the impact of dilatons in four-dimensional gauge theory we now look into the problem how a light dilaton modifies the Coulomb potential and its non-abelian analog [36]. It turns out that the dilaton introduces an ambiguity due to different boundary conditions which can be imposed on the dilaton: Two interesting solutions which arise include a regularized potential proportional to $(r + r_\phi)^{-1}$, where r_ϕ is inverse proportional to the decay constant of the dilaton, and a confining potential proportional to r .

Here we are interested in low energy gauge theories, i.e. in the dynamics of initially massless modes from the point of view of string theory. Since the compactification scale or string scale are many orders of magnitude larger than the weak scale, where the low energy degrees of freedom described in the standard model of particle physics acquire their masses, we do not expect the dilaton to couple to the relevant masses at the weak scale. Modulo an effective potential which the dilaton may have acquired on the road down from the string/compactification scales to temperatures below the SUSY scale, the influence of a dilaton on a low energy gauge theory is then described by a Lagrange density

$$\mathcal{L} = -\frac{1}{4} \exp\left(\frac{\phi}{f_\phi}\right) F_{\mu\nu}^j F^{\mu\nu}_j - \frac{1}{2} \partial^\mu \phi \cdot \partial_\mu \phi + \sum_{f=1}^{N_f} \bar{\psi}_f (i\gamma^\mu \partial_\mu + q\gamma^\mu A_\mu^j X_j - m_f) \psi_f, \quad (29)$$

with X_j denoting a defining N_c -dimensional representation of $\text{su}(N_c)$.

I already set the axion to zero, since the static pointlike source considered below does not excite the axion field.

The equations of motion are

$$\partial_\mu \left(\exp\left(\frac{\phi}{f_\phi}\right) F^{\mu\nu}_i \right) + q \exp\left(\frac{\phi}{f_\phi}\right) A_\mu^j f_{ij}^k F^{\mu\nu}_k = -q \bar{\psi} \gamma^\nu X_i \psi, \quad (30)$$

$$\partial^2 \phi = \frac{1}{4f_\phi} \exp\left(\frac{\phi}{f_\phi}\right) F_{\mu\nu}^j F^{\mu\nu}_j, \quad (31)$$

$$(i\gamma^\mu \partial_\mu + q\gamma^\mu A_\mu^j X_j - m) \psi = 0, \quad (32)$$

where here and in the sequel flavor indices are suppressed.

To analyze eq. (30) we will find it convenient to rewrite it in terms of the chromo-electric and magnetic fields $E_i = -F_{0i}^j X_j$, $B^i = \frac{1}{2} \epsilon^{ijk} F_{jk}^l X_l$:

$$\nabla \cdot \left(\exp\left(\frac{\phi}{f_\phi}\right) \mathbf{E} \right) - iq \exp\left(\frac{\phi}{f_\phi}\right) (\mathbf{A} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{A}) = \varrho,$$

$$\begin{aligned}
\partial_0 \left(\exp\left(\frac{\phi}{f_\phi}\right) \mathbf{E} \right) - \nabla \times \left(\exp\left(\frac{\phi}{f_\phi}\right) \mathbf{B} \right) + iq \exp\left(\frac{\phi}{f_\phi}\right) ([\Phi, \mathbf{E}] + \mathbf{A} \times \mathbf{B} + \mathbf{B} \times \mathbf{A}) &= -\mathbf{j} \\
\partial_0 \mathbf{B} + iq[\Phi, \mathbf{B}] + \nabla \times \mathbf{E} - iq(\mathbf{A} \times \mathbf{E} + \mathbf{E} \times \mathbf{A}) &= 0, \\
\nabla \cdot \mathbf{B} - iq(\mathbf{A} \cdot \mathbf{B} - \mathbf{B} \cdot \mathbf{A}) &= 0,
\end{aligned}$$

where in the gauge theory above $\varrho = q(\psi^+ \cdot X_k \cdot \psi)X^k$, $j_i = q(\bar{\psi} \cdot \gamma_i X_k \cdot \psi)X^k$, and we have included the Bianchi identities. In this section we use the letter Φ for A^0 .

To discuss the impact of the dilaton on the Coulomb potential we consider static configurations: $\partial_0 \varrho = 0$, $\mathbf{j} = 0$. Then we learn from $\partial_\mu j^\mu - iq[A_\mu, j^\mu] = 0$ that Φ and ϱ are in the same Cartan subalgebra: $[\Phi, \varrho] = 0$.

Pointlike stationary charge distributions, which in the present setting give rise to the generalized Coulomb potentials, are special cases of $SU(N_c)$ currents of the form

$$j^\mu(x) = \varrho^i(\mathbf{r}) X_i \eta^\mu_0 = \rho(\mathbf{r}) C^i X_i \eta^\mu_0 \quad (33)$$

carrying the same \mathbf{r} -dependence along any direction in color space. Such distributions arise for separable quark wave functions $\psi(x) = \varphi(x)\zeta$, where ζ is a constant Lorentz scalar in a spinor representation of $SU(N_c)$, and $\varphi(x)$ is a $SU(N_c)$ -invariant Dirac spinor whose left and right handed components differ only by a phase. We also assume both factors normalized according to $\int d^3\mathbf{r} \bar{\varphi} \cdot \varphi = 1$, $\zeta^+ \cdot \zeta = 1$.

For $SU(N_c)$ charges of the form (33) the vector potential can consistently be neglected, whence $\mathbf{E} = -\nabla\Phi$ and the Yang–Mills equations reduce to

$$\begin{aligned}
\nabla \cdot \left(\exp\left(\frac{\phi}{f_\phi}\right) \nabla \Phi \right) &= -\varrho, \\
[\Phi, \nabla \Phi] &= 0.
\end{aligned}$$

Due to (33) the second equation is fulfilled as a consequence of the first equation.

Our aim is to determine the chromo–electric potential for a point charge

$$\varrho_i(\mathbf{r}) = q C_i \delta(\mathbf{r})$$

where C_i denotes the expectation value of the generator X_i in color space. From the relation

$$(X_i)_{ab} (X^i)_{cd} = \frac{1}{2} \delta_{ad} \delta_{bc} - \frac{1}{2N_c} \delta_{ab} \delta_{cd} \quad (34)$$

one finds for arbitrary color content

$$\sum_{i=1}^{N_c^2-1} C_i^2 = \frac{N_c - 1}{2N_c}.$$

We thus want to determine the field of a stationary pointlike quark from

$$\nabla \cdot \left(\exp\left(\frac{\phi(\mathbf{r})}{f_\phi}\right) \mathbf{E}_i(\mathbf{r}) \right) = q C_i \delta(\mathbf{r}), \quad (35)$$

$$\nabla \times \mathbf{E}_i(\mathbf{r}) = 0, \quad (36)$$

and

$$\Delta\phi(\mathbf{r}) = -\frac{1}{2f_\phi} \exp\left(\frac{\phi(\mathbf{r})}{f_\phi}\right) \mathbf{E}_i(\mathbf{r}) \cdot \mathbf{E}^i(\mathbf{r}). \quad (37)$$

The unique radially symmetric solution to (35) can be written down immediately:

$$\exp\left(\frac{\phi(r)}{f_\phi}\right) \mathbf{E}_i(\mathbf{r}) = \exp\left(\frac{\phi(r)}{f_\phi}\right) E_i(r) \mathbf{e}_r = \frac{qC_i}{4\pi r^2} \mathbf{e}_r \quad (38)$$

whence equation (36) is also satisfied. Equation (37) then translates into

$$\frac{d^2}{dr^2} \phi(r) + \frac{2}{r} \frac{d}{dr} \phi(r) = -\frac{q^2}{64\pi^2 f_\phi} \left(1 - \frac{1}{N_c}\right) \exp\left(-\frac{\phi(r)}{f_\phi}\right) \frac{1}{r^4}. \quad (39)$$

The form of this equation suggests an ansatz $\frac{\phi(r)}{f_\phi} = a \ln\left(\frac{r}{b}\right)$, which yields the solution discussed below. However, we can solve (39) for arbitrary boundary conditions through a substitution

$$\xi = \frac{q}{4\pi f_\phi r} \sqrt{\frac{1}{2} - \frac{1}{2N_c}}, \quad \theta(\xi) = \frac{\phi(r)}{f_\phi}, \quad (40)$$

yielding⁷

$$\frac{d^2}{d\xi^2} \theta(\xi) = -\frac{1}{2} \exp(-\theta(\xi)), \quad (41)$$

or in terms of boundary conditions at infinity:

$$\theta'(\xi)^2 - \theta'(0)^2 = \exp(-\theta(\xi)) - \exp(-\theta(0)), \quad (42)$$

$$\xi = \int_{\theta(0)}^{\theta(\xi)} \frac{d\theta}{\sqrt{\exp(-\theta) - \exp(-\theta(0)) + \theta'(0)^2}},$$

where a sign ambiguity has been resolved by the requirement that the dilaton should not diverge at finite radius. The integral can be done elementary, with two branches depending on the sign of $\theta'(0)^2 - \exp(-\theta(0))$.

⁷We can map the dilaton equation of motion for arbitrary number d of spatial dimensions to eq. (41) through the substitution

$$\xi = \frac{q}{f_\phi} \sqrt{\frac{1}{2} - \frac{1}{2N_c}} G_d(r)$$

with

$$G_d(r) = -\frac{1}{2\pi} \ln\left(\frac{r}{r_0}\right), \quad d = 2,$$

$$G_d(r) = \frac{\Gamma(\frac{d}{2})}{2(d-2)\sqrt{\pi}^d} \frac{1}{r^{d-2}}, \quad d > 2.$$

The presence of the dilaton introduced a two-fold ambiguity in the Coulomb problem, and we have to determine from physical requirements which boundary conditions to chose.

For a first solution we require that the dilaton generated by the pointlike quark vanishes at infinity while the gradient satisfies the minimality condition

$$\lim_{r \rightarrow \infty} r^2 \frac{d}{dr} \phi(r) = -\frac{q}{4\pi} \sqrt{\frac{1}{2} - \frac{1}{2N_c}}. \quad (43)$$

This gives minimal kinetic energy for the dilaton at infinity subject to the constraint that the chromo-electric field does not develop a singularity for positive finite r . Then we find for the radial dependence of the dilaton and the electric field

$$\phi(r) = 2f_\phi \ln \left(1 + \frac{q}{8\pi f_\phi r} \sqrt{\frac{1}{2} - \frac{1}{2N_c}} \right), \quad (44)$$

$$\mathbf{E}_i(\mathbf{r}) = \frac{qC_i}{4\pi \left(r + \frac{q}{8\pi f_\phi} \sqrt{\frac{1}{2} - \frac{1}{2N_c}} \right)^2} \mathbf{e}_r, \quad (45)$$

implying a modified Coulomb potential

$$\Phi_i(r) = \frac{qC_i}{4\pi r + \frac{q}{2f_\phi} \sqrt{\frac{1}{2} - \frac{1}{2N_c}}}. \quad (46)$$

The result for gauge group $U(1)$ is received through the substitution $N_c \rightarrow -1$, and the corresponding dilaton-photon configuration was proposed already as a solitonic solution in a remarkable paper by Cvetič and Tseytlin [25].

The removal of the short distance singularity in the chromo-electric field would imply finite energy of the dilaton-gluon configuration:

$$E = \int d^3\mathbf{r} \left(\frac{1}{2} \nabla \phi \cdot \nabla \phi + \frac{1}{2} \exp\left(\frac{\phi(\mathbf{r})}{f_\phi}\right) \mathbf{E}_i(\mathbf{r}) \cdot \mathbf{E}^i(\mathbf{r}) \right) = 2qf_\phi \sqrt{\frac{1}{2} - \frac{1}{2N_c}}. \quad (47)$$

This regularization of the Coulomb potential at high energies is a very attractive property: In view of the prediction (27) it means that the dilaton resolves pointlike singularities at the Planck scale, fitting very well with the interpretation of the dilaton as a low energy imprint of a theory of quantum gravity.

It is clear that even a dilaton photon coupling and a resulting regularization of the electromagnetic Coulomb potential at or below l_{Pl} would not contradict accelerator based “confirmations” of pointlike structure of electrons above $10^{-3} \text{ fm} \simeq 10^{16} l_{Pl}$, and similarly the dilaton would also not show up spectroscopically: Applied to the hydrogen atom, the regularization (46) would imply a shift of energy levels of order

$$\frac{\Delta E}{E} \simeq -\sqrt{\frac{\alpha_{em}}{8\pi}} \frac{l_{Pl}}{a_B} \simeq -2.6 \times 10^{-28}.$$

For comparison, the 1S–2S level splitting and the Rydberg constant are known with a relative precision of order 10^{-11} [4], and counted in terms of orders of magnitude a dilaton regularized electromagnetic Coulomb potential is as invisible in high energy physics experiments as it is spectroscopically.

However, besides (46) there exists another quite intriguing solution if we require that $\frac{1}{q^2} \exp(\frac{\phi}{f_\phi})$ is independent of q . This requirement arises naturally in string theory, since the non-perturbatively fixed expectation value of the dilaton itself is supposed to determine the coupling. In the action (29) this requirement amounts to the constraint that the solution should respect the scale invariance of the equations of motion under

$$\phi \rightarrow \phi + 2\eta f_\phi$$

$$A \rightarrow \exp(-\eta)A$$

$$q \rightarrow \exp(\eta)q$$

for constant η . Eqs. (40,42) then imply $\theta'(\xi)^2 = \exp(-\theta(\xi)) = 4\xi^{-2}$, yielding

$$\phi(r) = 2f_\phi \ln\left(\frac{q}{8\pi f_\phi r} \sqrt{\frac{1}{2} - \frac{1}{2N_c}}\right), \quad (48)$$

$$\mathbf{E}_i(\mathbf{r}) = \frac{32\pi f_\phi^2}{q} \frac{N_c}{N_c - 1} C_i \mathbf{e}_r. \quad (49)$$

This corresponds to an energy density

$$\mathcal{H}(\mathbf{r}) = 4 \frac{f_\phi^2}{r^2}$$

whence the energy in a volume of radius r diverges linearly:

$$E|_r = 16\pi f_\phi^2 r.$$

This is an infrared divergence, whence it should not be related to new physics at short distances, and it would cost an infinite amount of energy to create an isolated quark. Gauge theory with a dilaton thus accomodates both Coulomb and confining phases in a simple way.

6 The axidilaton and stabilization of the dilaton in four dimensions

Yet we have been missing the pseudo-scalar axion which couples to the instanton density $F \cdot \tilde{F}$. The motivation for including an axion in theories with a dilaton is four-fold: Historically the first and still a very important motivation for the axion arose from the observation that it explains the absence of a CP violating phase in gauge theories [77, 103, 110]. Besides this an axion arises also as a massless excitation of closed superstrings as a companion of the graviton and the dilaton [53], and it accompanies the dilaton in supersymmetric gauge theories: If the dilaton arises in the real part of the lowest component of a chiral superfield the corresponding imaginary part is an axion. Furthermore, under a certain constraint on decay constants the axion-dilaton system exhibits a duality symmetry commonly denoted as S-duality. This has been realized both in field theory [88] and in string theory⁸ [47, 84, 85, 87]. The dilaton and the axion are mixed under this symmetry in a non-linear way, and recent developments in string theory indicate that S-duality should be a generic feature of grand unified quantum field theories inherited from string theory.

The primary motivations for contemporary considerations of a dilaton arise from string theory, and in this spirit emphasis in the present discussion will also be on a string inspired axion. The difference does not show up in the coupling to gauge fields, but in the vacuum sector: A Peccei-Quinn type axion is an angular variable and has at most finitely many different vacua. The string axion on the other hand arises as a dual field to an antisymmetric tensor and has no reason to be periodic. It also will not necessarily couple to light fermion masses, yet it still suppresses a CP violating θ -angle in non-abelian gauge theory. To explain how this comes about, note that an axion explains absence of CP violation with or without Peccei-Quinn symmetry in the fermionic sector: If we temporarily include the axion scale in the axion $\Theta(x)$, which thus becomes dimensionless, the relevant axion-gluon term in the presence of θ is

$$\frac{q^2}{32\pi^2}(\Theta + \theta)F^{\mu\nu}{}_j\tilde{F}_{\mu\nu}{}^j.$$

However, instantons always induce an effective axion potential such that the vacuum expectation values of the axion satisfy $\langle\Theta\rangle + \theta = 2\pi n$ for some integer n , and this eliminates CP violation from the $F\tilde{F}$ -term. From this point of view Peccei-Quinn symmetry on the fermions only introduces an additional Higgs field, in order to derive the relevant interaction indirectly through an anomaly.

In the spirit of employing Kaluza-Klein theory as a paradigm for theories with a dilaton we first consider the axion from a five-dimensional point of view: In the presence of a five-dimensional threshold the axion should arise as the fifth component of a pseudo-vector.

⁸Target space duality mixes dilaton- and axion-like degrees of freedom in string theory in a similar manner, see [45, 46] and references there.

The Kaluza–Klein Ansatz (21) then yields

$$\frac{q^2}{64\pi^2}\sqrt{-G}\epsilon^{JKLMN}\Theta_J F_{KL}{}^j F_{MNj} = \frac{q^2}{64\pi^2}\sqrt{-g}\epsilon^{\mu\nu\rho\sigma}[\Theta F_{\mu\nu}{}^j F_{\rho\sigma j} + 4\Theta_\mu F_{\nu\rho j} D_\sigma A^j],$$

where $\epsilon_{01234} = -\sqrt{-G}$, i.e. ϵ is a tensor, not a density. Since this term transforms into a divergence under $\Theta_K \rightarrow \Theta_K + \partial_K \alpha$, it is natural to propose a Maxwell term as kinetic term:

$$-\frac{1}{4}\sqrt{-G}\Theta_{MN}\Theta^{MN} = -\frac{1}{4}\sqrt{-g}\Phi^{\frac{1}{3}}[b_{\mu\nu}b^{\mu\nu} + \frac{2}{\Phi}g^{\mu\nu}\partial_\mu\Theta \cdot \partial_\nu\Theta], \quad (50)$$

where $\Theta_{MN} = \partial_M\Theta_N - \partial_N\Theta_M$ and

$$b_{\mu\nu} = \Theta_{\mu\nu} + \sqrt{2\kappa}a_\mu\partial_\nu\Theta - \sqrt{2\kappa}a_\nu\partial_\mu\Theta.$$

Here we have rescaled the graviphoton $a_\mu \rightarrow \sqrt{2\kappa}a_\mu$ to have canonical mass dimension.

It is an interesting property of a five–dimensional threshold that the power of the dilaton in front of the kinetic term of the axion Θ is such that it matches exactly with the $\text{SL}(2, \mathbb{R})$ duality for the axidilaton system described below (55). This is a unique property of reductions from five to four dimensions and in remarkable coincidence with expectations from string theory.

In the sequel we will use the following pseudo–vector in four dimensions:

$$\begin{aligned} \bar{\Theta}_\mu &= \Theta_\mu - \sqrt{2\kappa}a_\mu\Theta, \\ \bar{\Theta}_{\mu\nu} &= b_{\mu\nu} - \sqrt{2\kappa}\Theta f_{\mu\nu}. \end{aligned}$$

In order to motivate the following investigations, we take a closer look at the action of the zero modes of five–dimensional Einstein–Yang–Mills theory with fermions compactified to four dimensions. We relate the dilaton ϕ to the metric coefficient Φ by

$$\Phi = \exp(\sqrt{6\kappa}\phi),$$

where the four–dimensional gravitational constant κ is the five–dimensional gravitational constant divided by the circumference of the compact dimension. After appropriate rescalings of the other fields and coupling constants, we infer the following action in four dimensions from the results of section 3:

$$\begin{aligned} \frac{1}{\sqrt{-g}}\mathcal{L} &= \frac{1}{2\kappa}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi \cdot \partial_\nu\phi - \frac{1}{4}\exp(\sqrt{6\kappa}\phi)f_{\mu\nu}f^{\mu\nu} \\ &\quad - \frac{1}{2}\exp\left(-\sqrt{\frac{8\kappa}{3}}\phi\right)g^{\mu\nu}(D_\mu A_j \cdot D_\nu A^j + \partial_\mu\Theta \cdot \partial_\nu\Theta) \\ &\quad - \frac{1}{4}\exp\left(\sqrt{\frac{2\kappa}{3}}\phi\right)[V_{\mu\nu}{}^j V^{\mu\nu}{}_j + \bar{\Theta}_{\mu\nu}\bar{\Theta}^{\mu\nu} + 2\sqrt{2\kappa}(V_{\mu\nu}{}^j A_j + \bar{\Theta}_{\mu\nu}\Theta)f^{\mu\nu} + 2\kappa(A_j A^j + \Theta^2)f_{\mu\nu}f^{\mu\nu}] \\ &\quad + \frac{q^2}{64\pi^2 f_a}\sqrt{-g}\epsilon^{\mu\nu\rho\sigma}\left(V_{\mu\nu}{}^j + \sqrt{2\kappa}(a_\nu D_\mu A^j - a_\mu D_\nu A^j + A^j f_{\mu\nu})\right) \end{aligned}$$

$$\times \left(\Theta [V_{\rho\sigma j} + \sqrt{2\kappa}(a_\sigma D_\rho A_j - a_\rho D_\sigma A_j + A_j f_{\rho\sigma})] + 4(\bar{\Theta}_\rho + \sqrt{2\kappa}a_\rho \Theta) D_\sigma A_j \right),$$

and each fermion species of mass M at the compactification scale contributes a term

$$\frac{1}{\sqrt{-g}} \mathcal{L}_f = \bar{\psi} e^\mu{}_a \gamma^a (i\partial_\mu + i\omega_\mu + qV_\mu) \psi - M \exp\left(-\sqrt{\frac{\kappa}{6}}\phi\right) \bar{\psi} \psi.$$

Again, as pointed out already in section 3, the graviphoton ensures invariance of the effective four-dimensional gauge fields under diffeomorphisms along the fifth dimension:

$$\begin{aligned} V_\mu{}^j &= A_\mu{}^j - \sqrt{2\kappa} a_\mu A^j \\ V_{\mu\nu}{}^j &= F_{\mu\nu}{}^j + \sqrt{2\kappa} (a_\mu D_\nu A^j - a_\nu D_\mu A^j - A^j f_{\mu\nu}). \end{aligned}$$

The classical equations of motion of the system above are invariant under constant shifts of the dilaton

$$\begin{aligned} \phi &\rightarrow \phi + c, \\ e^\mu{}_a &\rightarrow \exp\left(-\sqrt{\frac{\kappa}{6}}c\right) e^\mu{}_a, \\ a_\mu &\rightarrow \exp\left(-\sqrt{\frac{2\kappa}{3}}c\right) a_\mu, \\ V_\mu &\rightarrow V_\mu, \quad \bar{\Theta}_\mu \rightarrow \bar{\Theta}_\mu, \\ \psi &\rightarrow \exp\left(-\frac{1}{2}\sqrt{\frac{\kappa}{6}}c\right) \psi, \\ A &\rightarrow \exp\left(\sqrt{\frac{2\kappa}{3}}c\right) A, \\ \Theta &\rightarrow \exp\left(\sqrt{\frac{2\kappa}{3}}c\right) \Theta, \end{aligned} \tag{51}$$

since this just rescales the action according to

$$S \rightarrow \exp\left(\sqrt{\frac{2\kappa}{3}}c\right) S.$$

This symmetry can equivalently be formulated as a scaling symmetry on the co-ordinates, and the dilaton is often denoted as a Goldstone boson for dilatations. However, the symmetry is unbroken as long as the dilaton remains massless, and I prefer the modern designation of the dilaton as a flat direction.

The origin of the symmetry of the equations of motion under (51) is easily understood from the Kaluza–Klein origin of the action. The scale transformations are equivalent to a rescaling of the internal dimensions by a factor $\exp\left(\sqrt{\frac{2\kappa}{3}}c\right)$, and the equations of motion resulting from a Kaluza–Klein Ansatz do not carry any remembrance of the internal scale

since we neglected any massive modes related to ∂_5 . Therefore, we should not expect a perturbatively generated dilaton potential, since the perturbative dynamics of the Kaluza–Klein zero modes only depends on the fluctuations of the internal dimensions through the dilaton, but not on their actual size. However, if there exist inherently four–dimensional effects in the low energy dynamics, then we might expect a non–perturbatively generated dilaton potential, since unwinding of internal dimensions would certainly conflict with inherently four–dimensional effects. A genuine four–dimensional effect is the appearance of instantons in gauge theories, and therefore we will concentrate on the issue whether instantons create a dilaton potential. Indeed, we will find that instantons create a dilaton mass, because generically instantons imply that a small dilaton would be energetically favored, while the axions push the dilaton to large values.

This nicely complies with ideas about duality symmetries between axions and dilatons: Instantons create an effective axion potential and we have emphasized before that there emerged much evidence in recent years for a duality symmetry between the axion and the dilaton, which is described in eqs. (53–55) below. A fundamental axion acquiring an effective potential through instanton mediated tunneling effects thus provides a very strong indication for a light dilaton acquiring an effective potential in a similar vein.

From a Minkowski space point of view an instanton contribution to an effective dilaton potential may also be described as a gluon condensate, and given the no–go conjecture for a perturbative origin condensates provide a natural mechanism to generate terms in a dilaton potential. Besides a gluon condensate we may expect from the coupling to the kinetic energy of the axion a contribution from a condensate $\langle \partial a \cdot \partial a \rangle$ [34, 35], or from a gluino condensate [30, 38, 94], and we will take a very brief look at a gluino condensate in the next section. Recent discussions of contributions from gluino condensates can be found in [67], where the coupling of the chiral dilaton multiplet is re–examined, and in [14], where the dilaton is treated in the linear multiplet. The proposal of a self–dual coupling of the axidilaton to the gluons is reviewed in [75].

In the present section we will concentrate on the contribution from the axions and its implications. Axions provide an attractive new mechanism for dilaton stabilization since the exponents of the dilaton multiplying the gluon and axion terms differ in sign, and since non–trivial axion configurations provide suggestive explanations for a condensate $\langle \partial a \cdot \partial a \rangle$. This alternative proposal for generation of a dilaton potential implies a drastic change of scales: While a gluino condensate would be expected to generate a dilaton potential at the SUSY breaking scale around or above 1 TeV, the axion would stabilize the dilaton at the QCD scale around 1 GeV.

In the sequel the notation for the axion will be changed $\Theta \rightarrow a$, since graviphotons will be neglected and a is a more standard notation for the axion in four–dimensional field theory. The main players in the game are then the dilaton ϕ , the axion a and gauge fields A_μ with field strengths $F_{\mu\nu}$, and their mutual interactions before taking into account non–perturbative effects are governed by the Lagrangian

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2\kappa}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi \cdot \partial_\nu\phi - \frac{1}{2}\exp(-2\frac{\phi}{f_\phi})g^{\mu\nu}\partial_\mu a \cdot \partial_\nu a \quad (52)$$

$$-\frac{1}{4}\exp(\frac{\phi}{f_\phi})F_{\mu\nu}^j F^{\mu\nu}_j + \frac{q^2}{64\pi^2 f_a}\epsilon^{\mu\nu\rho\sigma}aF_{\mu\nu}^j F_{\rho\sigma j}.$$

The dilaton–axion and dilaton–gluon couplings in (52) match in such a way that the system exhibits an $\text{SL}(2, \mathbb{R})$ duality symmetry, or S–duality for short, if the scales are related by

$$f_\phi = \frac{8\pi^2}{q^2} f_a. \quad (53)$$

The invariance of the equations of motion under the duality transformations is most conveniently described in terms of the axidilaton

$$z = \frac{1}{f_a} [a + i f_\phi \exp(\frac{\phi}{f_\phi})] \quad (54)$$

and the symmetry is realized via

$$z' = \frac{a_{11}z + a_{12}}{a_{21}z + a_{22}}, \quad a_{11}a_{22} - a_{12}a_{21} = 1, \quad (55)$$

$$F'_{\mu\nu} - i\tilde{F}'_{\mu\nu} = (a_{21}z + a_{22})(F_{\mu\nu} - i\tilde{F}_{\mu\nu}),$$

which means that the self–dual part of the Yang–Mills curvature transforms like a half–differential on the axidilaton upper half–plane.

The invariance of equations of motion plus Bianchi identities is easily recognized if the equations of motion are written as

$$\frac{\partial^2 z}{(z - \bar{z})^2} - 2 \frac{\partial z \cdot \partial \bar{z}}{(z - \bar{z})^3} = \frac{f_a}{32i f_\phi^3} (F + i\tilde{F})^2,$$

$$D_\mu \text{Im}\{z(F^{\mu\nu}_j - i\tilde{F}^{\mu\nu}_j)\} = 0,$$

where ∂^2 denotes the covariant Laplacian.

The scaling symmetry in (55) in the abelian case is similar to but different from the rescaling (51): $\delta a = 2\varepsilon a$, $\delta\phi = 2\varepsilon f_\phi$, $\delta A_\nu = -\varepsilon A_\nu$, and it implies a Noether current

$$\frac{1}{\sqrt{-g}} j^\mu_\phi = f_\phi g^{\mu\nu} \partial_\nu \phi + a \exp(-2\frac{\phi}{f_\phi}) g^{\mu\nu} \partial_\nu a - \frac{1}{2} \exp(\frac{\phi}{f_\phi}) F^{\mu\nu} A_\nu + \frac{q^2}{16\pi^2} \frac{a}{f_a} \tilde{F}^{\mu\nu} A_\nu. \quad (56)$$

Both in abelian and non–abelian theories the Peccei–Quinn symmetry $z \rightarrow z + a_{12}$ leaves the gauge potentials invariant and yields a conserved current

$$\frac{1}{\sqrt{-g}} j^\mu_a = f_a \exp(-2\frac{\phi}{f_\phi}) g^{\mu\nu} \partial_\nu a + \frac{q^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} (A_\nu^j \partial_\rho A_{\sigma j} + \frac{q}{3} f_{ijk} A_\nu^i A_\rho^j A_\sigma^k), \quad (57)$$

and the scaling symmetry (51) of the equations of motion is also preserved with $2\kappa \rightarrow 3f_\phi^{-2}$.

In view of the currents (56,57) the designation of f_ϕ and f_a as decay constants looks very natural and suggestive, since the scales parametrize non–vanishing matrix elements

between the (pseudo-)scalars and the vacuum, similar to the pion decay constant. However, there is an important difference which should be kept in mind: The pion decay constant parametrizes matrix elements $\langle 0 | j_\pi^\mu | \pi \rangle$ which actually contribute to pion decays into lepton pairs through intermediate vector bosons, and the matrix element arises in the microscopic theory at low energies when the leptonic sector of the 4-Fermi-vertex has already been evaluated. Nothing like that is expected to take place for a fundamental axidilaton in string theory or Kaluza-Klein theory, and the scales f_ϕ and f_a appear only with negative powers in physical matrix elements.

Yet we have not taken into account non-trivial field configurations of the gauge fields and the axion: We infer the non-perturbative effects of these field configurations from the Lagrangian of the Euclidean action. In a flat background this takes the form:

$$\begin{aligned} \mathcal{L}_E = & \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \cdot \partial_\nu \phi + \frac{1}{2} \exp(-2\frac{\phi}{f_\phi}) g^{\mu\nu} \partial_\mu a \cdot \partial_\nu a \\ & + \frac{1}{4} \exp(\frac{\phi}{f_\phi}) F_{\mu\nu}^j F^{\mu\nu}_j - i \frac{q^2}{32\pi^2 f_a} a \tilde{F}_{\mu\nu}^j F^{\mu\nu}_j. \end{aligned} \quad (58)$$

Positivity of the real part and the estimate of the effective axion potential by Vafa and Witten [97] indicate that the dominating contributions to the path integral come from instanton configurations $F = \pm \tilde{F}$ with constant dilaton and the axion frozen to integer multiples of $2\pi f_a$. This survival of instantons in the presence of the dilaton is crucial, since integrality of the instanton number and invariance of the path integral discretize Peccei-Quinn symmetry

$$\frac{a_{12}}{2\pi} \in \mathbb{Z},$$

thereby also breaking the scale invariance (51).

The impact of instantons on the effective axion potential has been examined by several authors, and the interpretation of instantons as real time tunneling configurations between gauge theory vacua suggests

$$V(a) = m_a^2 f_a^2 \left(1 - \cos\left(\frac{a}{f_a}\right) \right) \quad (59)$$

if the instanton gas is dilute enough to neglect higher order cosine terms [23, 57, 63]. While initially this result was inferred from semiclassical calculations of tunneling amplitudes, the same potential can also be derived in a direct instanton calculation if the wavelength of the axion is large compared to the instanton size.

The picture emerging from this shows us that instantons create an effective axion potential with an enumerable set of equidistant vacua, thus discretizing Peccei-Quinn symmetry. Discreteness of the axion vacua and the cosine-like shape of the axion in turn breaks the scale invariance (51) and indicates that the axidilaton-gluon system also lifts the degeneracy of the dilaton. This is obvious in the gauge sector: Instantons push the dilaton into the strong gauge coupling regime, since the action of the instantons decreases with decreasing $\langle \phi \rangle$. However, non-trivial configurations also arise in the axion sector:

- If a is periodic $a \sim a + 2\pi f_a$, then it contributes non-trivial configurations to the Euclidean path integral over $\exp(-S_E)$ in the form of *axion walls* (instead of axion strings in three dimensions). Periodicity of a arises, if it is related to the argument of a complex field with frozen modulus in the low energy regime. This is e.g. the case if a arises as the phase of a determinant of local fermion masses.
- If a is not an angular variable, then all the possible vacua $\langle a \rangle = 2\pi f_a n$ are distinct and we expect three-dimensional domain walls separating four-dimensional domains where a approximates different vacua.

String theory is essential for the stability of these defects, since the four axion scattering amplitude at string tree level depends non-trivially on the momenta of the scattered axions [83], whence (52) contains only lowest order terms in a derivative expansion, as expected for an effective low energy theory. However, we will not attempt a systematic derivation of the higher order derivative terms from string scattering amplitudes, but rather adopt a phenomenological approach in borrowing methods from the theory of cosmic strings to estimate the axion condensate.

Both kinds of topological defects mark regions of non-vanishing gradients ∂a and favor large values of the dilaton through the dilaton-axion coupling, thus compensating the effect of the instantons. Therefore, we expect an effective dilaton potential cutting off large values of the dilaton through an average background field strength, while small values of the dilaton are suppressed by the variance of the axion. After adjustment of q the potential results:

$$V(\phi) = \frac{m_\phi^2 f_\phi^2}{6} \left(2 \exp\left(\frac{\phi}{f_\phi}\right) + \exp\left(-2\frac{\phi}{f_\phi}\right) \right). \quad (60)$$

In the resulting model the dilaton mass m_ϕ and the coupling constant q parametrize the background field strength from the instantons and the axion gradients from domain boundaries.

We may give estimates on q and m_ϕ in terms of an average instanton scale ϱ and a characteristic length Δ of the axion defects. In the case of an angular axion Δ would measure the circumference of the axion walls, while in the case of vacuum domains of the axion the four-dimensional domain boundaries are extended in three dimensions and have an average thickness Δ in the fourth direction. The average separation ϱ of instantons in the instanton liquid is about three times larger than the average extension of the instantons [90, 82]. From this we find an estimate for the effective dilaton potential

$$V(\phi) = \frac{16}{q^2 \varrho^4} \exp\left(\frac{\phi}{f_\phi}\right) + 2\pi^2 \frac{f_a^2}{\Delta^2} \exp\left(-2\frac{\phi}{f_\phi}\right). \quad (61)$$

This implies for the gauge coupling and the dilaton mass

$$\frac{1}{q} = \frac{\pi f_a \varrho^2}{2\Delta}, \quad (62)$$

$$m_\phi f_\phi = \frac{4\sqrt{3}}{q \varrho^2}. \quad (63)$$

This investigation can be pursued further if a is not an angular variable: In this case we may estimate the parameter Δ by minimizing the energy density of the axion domain boundaries

$$u = 2\pi^2 \frac{f_a^2}{\Delta} + m_a^2 f_a^2 \Delta \quad (64)$$

yielding a thickness of the order

$$\Delta \simeq \frac{\pi\sqrt{2}}{m_a} \quad (65)$$

which is of the same order as the thickness of ordinary axion domain walls in Minkowski space [64]. From (62) and (65) we find a relation between the axion parameters and the average instanton radius

$$m_a^2 f_a^2 \simeq \frac{2}{\pi\alpha_q \varrho^4}. \quad (66)$$

The average instanton radius at low temperatures is approximately [90]

$$\frac{1}{\varrho} \simeq 200 \text{ MeV}$$

and the big uncertainty in estimating axion parameters from (66) concerns the coupling constant at the scale where instanton mediated tunneling induces the axion mass. In QCD the investigation of a truncated Dyson–Schwinger equation for the gluon propagator by Alkofer, Hauck and von Smekal indicates a value $\alpha_q \sim O(10)$ [2], and with the previous results this points at a value

$$m_a f_a \sim 10^4 \text{ MeV}^2. \quad (67)$$

Although we did not use any information or assumptions about axion–fermion couplings, it is remarkable that (67) complies with current algebra based estimates for an axion coupling to light quarks [9, 62, 92, 51, 10]:

$$m_a f_a \sim m_\pi f_\pi.$$

The potential (61) and eq. (66) imply a relation between the dilaton mass and the axion mass

$$m_\phi f_\phi \simeq \sqrt{6} m_a f_a. \quad (68)$$

Since S–duality will certainly be broken at the scales under consideration we expect a dilaton decay constant of the order of the Planck mass while f_a could be in the phenomenologically preferred range $10^{10} - 10^{12}$ GeV. Compared to the axion this implies a much smaller mass of the dilaton and a later onset of coherent dilaton oscillations.

7 The supersymmetric theory

The supersymmetric framework is of interest both from up–down and bottom–up approaches to physics beyond the standard model of particle physics. On the one hand on an intermediate energy scale below the compactification scale string theory predicts that the relevant physical degrees of freedom should be described by a supergravity theory. On the other hand supersymmetric gauge theories evolved into a primary tool for model building beyond the standard model, and a footprint of supersymmetry is considered as one of the most spectacular results that might be expected from accelerator physics on foreseeable time scales. The subject got a further boost a few years ago by the approximate convergence of coupling constants around 10^{16} GeV if supersymmetry begins to apply at the TeV–scale⁹ [44, 52, 3, 65].

Given the necessity to introduce an axion to solve the strong CP problem, supersymmetry provides an independent motivation to also introduce a dilaton, since an axion in a chiral superfield always comes with a dilaton [112, 31].

With a few notational changes we will follow the conventions of Wess and Bagger [106]. Supersymmetry is conveniently described in terms of superfields. These are Grassmann valued fields over space–time, where the Grassmann algebra is generated by a constant Dirac spinor θ of mass dimension $-\frac{1}{2}$:

$$\{\theta_\alpha, \theta_\beta\} = 0, \quad \{\theta_\alpha, \bar{\theta}_\beta\} = 0, \quad \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = 0.$$

The particular superfields which we need are the chiral dilaton multiplet

$$S = d + i\theta \cdot \sigma^\mu \cdot \bar{\theta} \partial_\mu d + \frac{1}{4}\theta^2 \bar{\theta}^2 \partial^2 d + \sqrt{2}\theta \cdot \delta - \frac{i}{\sqrt{2}}\theta^2 \partial_\mu \delta \cdot \sigma^\mu \cdot \bar{\theta} + \theta^2 Z$$

and the spinorial chiral superfield containing the gauge fields:

$$\begin{aligned} W_\alpha = & -i\lambda_\alpha + \theta \cdot \sigma^\mu \cdot \bar{\theta} \partial_\mu \lambda_\alpha - \frac{i}{4}\theta^2 \bar{\theta}^2 \partial^2 \lambda_\alpha + \theta_\alpha D - \frac{i}{2}\theta^2 \sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_\mu D \\ & + \frac{i}{2}(\sigma^\mu \cdot \bar{\sigma}^\nu)_\alpha{}^\beta \theta_\beta F_{\mu\nu} + \frac{1}{4}\theta^2 (\sigma^\mu \cdot \bar{\sigma}^\nu \cdot \sigma^\rho)_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_\rho F_{\mu\nu} + \theta^2 \sigma^\mu_{\alpha\dot{\alpha}} (\partial_\mu \bar{\lambda}^{\dot{\alpha}} - iq[A_\mu, \bar{\lambda}^{\dot{\alpha}}]). \end{aligned}$$

The lowest component d of the dilaton multiplet is related to the axidilaton (54) through

$$d = i \frac{q^2}{16\pi^2} \bar{z},$$

and we have normalized the multiplet such that it has mass dimension zero, i.e. the dilatino δ has mass dimension $\frac{1}{2}$. Note that our gauge field A_μ and the gauge fields v_μ in [106] differ in sign. The gluino λ is the fermionic superpartner of the gluon.

⁹This analysis is continuously improved, and predictions for masses of supersymmetric particles are to a large degree model dependent [6, 66].

The dilaton gluon coupling arises as the real part of the θ^2 -component of SW^2 :

$$SW_j^\alpha W_\alpha^j|_{\theta^2} = -\lambda_j^\alpha \lambda_\alpha^j Z + i\sqrt{2}\delta^\alpha \lambda_\alpha^j D_j - \frac{1}{\sqrt{2}}(\sigma^\mu \cdot \bar{\sigma}^\nu)_\alpha^\beta F_{\mu\nu}^j \lambda_j^\alpha \delta_\beta \quad (69)$$

$$+d[D^j D_j + i(D_\mu \lambda)^j \cdot \sigma^\mu \cdot \bar{\lambda}_j - i\lambda_j \cdot \sigma^\mu \cdot (D_\mu \bar{\lambda})^j - \frac{1}{2}F_{\mu\nu}^j F^{\mu\nu}_j - \frac{i}{2}\tilde{F}_{\mu\nu}^j F^{\mu\nu}_j],$$

and Witten observed that the correct supersymmetrization of kinetic terms of the axidilaton is given by a logarithm of superfields [112]:

$$\ln(S + S^+)|_{\theta^2\bar{\theta}^2} = \frac{1}{(d + d^+)^2}(\partial d^+ \cdot \partial d + \frac{i}{2}\delta \cdot \sigma^\mu \cdot \partial_\mu \bar{\delta} - \frac{i}{2}\partial_\mu \delta \cdot \sigma^\mu \cdot \bar{\delta} - Z^+ Z). \quad (70)$$

Neglecting the quarks and squarks, the supersymmetrization of (52) is then given by

$$\mathcal{L} = -2f_\phi^2 \ln(S + S^+)|_{\theta^2\bar{\theta}^2} + \frac{1}{2}SW_j^\alpha W_\alpha^j|_{\theta^2} + \frac{1}{2}S^+ \bar{W}_j^{\dot{\alpha}} \bar{W}_{\dot{\alpha}}^j|_{\bar{\theta}^2} \quad (71)$$

if the decay constants satisfy the self-duality condition (53).

In passing we also note that there exist numerous possibilities to supersymmetrize the effective axion potential (59), e.g. through a superpotential

$$\mathcal{V} = 2m_a f_a^2 \cosh\left(\frac{f_\phi}{f_a}\left(S - \frac{1}{2}\right)\right).$$

However, since $m_a f_a \ll \Lambda_{SUSY}^2$ instantons will not dominate the gauge theory vacuum above the scale of supersymmetry breaking, and we will not pursue these superpotentials further.

With eqs. (68) and (71) at hand we may now also provide an estimate on the lower bound of values of the dilaton decay constant for which our approximation of dominance of the axion condensate is applicable:

Elimination of the auxiliary field Z of the dilaton multiplet yields the dilaton gluon coupling

$$\mathcal{L}_{\lambda\phi} = -\frac{1}{8f_\phi^2} \exp(2\frac{\phi}{f_\phi}) \bar{\lambda}^2 \lambda^2.$$

If there is a gluino condensate with a scale $\Lambda_{SUSY} \sim 1$ TeV, this would contribute a dilaton mass term¹⁰ $m_\phi^\lambda \simeq \Lambda_{SUSY}^3 f_\phi^{-2}$. As a consequence the variance of the axion would dominate the dilaton mass at scales where instanton induced tunneling becomes relevant for dilaton decay constants above 10^{11} GeV, while for decay constant below this value the gluino condensate would dominate to very low temperatures.

¹⁰Note that restoration of the dilaton decay constant shows that a dilaton mass from gluino condensates would be tiny, too. Therefore the dilaton would also provide a suitable candidate for cold dark matter in the framework of supersymmetry breaking.

8 The dilaton as a dark matter candidate

We have seen that both instanton tunneling and supersymmetry breaking favor a very light weakly coupled dilaton accompanying a light weakly coupled axion. However, light weakly coupled (pseudo-)scalars (or scalars for short) are generically expected to make an appreciable contribution to the energy density of the universe in the form of cold dark matter, which in this specific instance means that they developed coherent oscillations with non-relativistic momenta. The onset of coherent oscillations is expected when the universe has cooled down to temperatures where mass terms begin to dominate over dissipative expansion terms in the equations of motion of the scalars. Coherent oscillations are then expected to dominate the energy density of the scalars, since due to the weak coupling thermal creation and annihilation of the scalars can be neglected.

There exist several monographs where the general background for cosmology and the impact of particle physics is very well presented, see e.g. [15, 64, 79]. However, I will begin with a review of a few basic facts to set the stage for the discussion of the role the dilaton.

In discussing cosmological implications of the dilaton we will stick to the usual approximation of spatial homogeneity and isotropy, i.e. we will discuss dynamics in a Robertson–Walker space–time with line element

$$ds^2 = -dt^2 + \mathcal{R}^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\vartheta^2 + r^2 \sin^2 \vartheta d\varphi^2 \right), \quad (72)$$

where r is dimensionless and the scale factor \mathcal{R} has the dimension of a length. For $k = 1$ the spatial part of the metric can be described as a 3–sphere of radius $\mathcal{R}(t)$ in flat \mathbb{R}^4 , while $k = -1$ is the corresponding hyperbolic space. $k = 0$ is flat 3–space.

The form (72) of the metric implies that the matter energy momentum tensor has the form $T_{00} = \varrho(t)$, $T_{0j} = 0$, $T_{ij} = p(t)g_{ij}$, and that energy conservation $\nabla_\mu T^{\mu 0} = 0$ reads

$$\frac{d}{dt}(\varrho \mathcal{R}^3) = -p \frac{d}{dt}(\mathcal{R}^3). \quad (73)$$

If particle interactions are fast enough to maintain thermal equilibrium during expansion, the first and second law of thermodynamics in a system with N_i particles of chemical potential μ_i

$$dE = TdS - pdV + \sum_i \mu_i dN_i$$

implies due to eq. (73)

$$\frac{dS}{dt} = - \sum_i \frac{\mu_i}{T} \frac{dN_i}{dt}, \quad (74)$$

where S and N_i are the entropy and particle numbers in a unit of comoving volume $v = V\mathcal{R}^{-3}$.

I would like to add a remark on the thermodynamical expression for the energy density appearing in the energy momentum tensor, since there exists some confusion about this central object:

There are many ways to divide a thermodynamic potential by a volume, but the energy density T_{00} is

$$\varrho = \left. \frac{\partial E}{\partial V} \right|_t.$$

However, in a FRW universe temperatures and chemical potentials will only depend on t , while particle numbers also go with the volume. From this we may immediately translate the expression for ϱ into thermodynamics:

$$\varrho = \left. \frac{\partial E}{\partial V} \right|_{T,\mu} = T \left. \frac{\partial S}{\partial V} \right|_{T,\mu} - p + \mu \cdot \left. \frac{\partial N}{\partial V} \right|_{T,\mu},$$

with an obvious abbreviation for the sum over particle species. From the grand potential $\Omega_{GC}(T, V, \mu) = -pV$, $d\Omega_{GC} = -SdT - pdV - N \cdot d\mu$ we learn that

$$\left. \frac{\partial S}{\partial V} \right|_{T,\mu} = \left. \frac{\partial p}{\partial T} \right|_{V,\mu},$$

and that the particle densities are

$$\nu_i \equiv \left. \frac{\partial N_i}{\partial V} \right|_{T,\mu} = \left. \frac{\partial p}{\partial \mu_i} \right|_{T,V,\hat{\mu}_i},$$

where $\hat{\mu}_i = \mu_1, \dots, \mu_{i-1}, \mu_{i+1}, \dots$. With $\beta = \frac{1}{kT}$ a useful expression for ϱ is then

$$\varrho = T \left. \frac{\partial p}{\partial T} \right|_{V,\mu} - p + \mu \cdot \nu = \mu \cdot \nu - \left. \frac{\partial}{\partial \beta} (\beta p) \right|_{V,\mu}, \quad (75)$$

because this directly relates ϱ to Ω_{GC} .

In the case of one particle species the dynamical evolution would then be determined as follows: The energy levels of the particles determine the grand potential and the pressure $p(T, \mu)$. Eq. (75) is then used to calculate $\varrho(T, \mu)$, while in the thermodynamic limit the entropy and particle number in a comoving volume are $S = \frac{\partial p}{\partial T} \mathcal{R}^3$, $N = \frac{\partial p}{\partial \mu} \mathcal{R}^3$. Equations (73,74) and the Friedmann equation (76) below then constitute a set of three first order differential equations for the dynamical variables $\mathcal{R}(t)$, $T(t)$ and $\mu(t)$.

The algebra of dynamical degrees of freedom in a quasi-statically expanding FRW model is therefore generated by the scale parameter which describes expansion or contraction with a relative velocity $H = \frac{d}{dt} \ln(\mathcal{R})$, the temperature T and the chemical potentials of the various particle species. Of course, in practice equilibrium is not maintained for weakly interacting particle species which decouple due to thinning out in the expanding universe. This is taken care of by assigning extra effective temperatures governing the energy distribution within these particles species. Furthermore, it proved a very useful approximation to calculate the history of the homogeneous and isotropic background piecing together different epochs where the energy content of the universe was stored primarily either in radiation, in pressureless matter, or in scalar fields. In the first two cases the

relevant degrees of freedom for the evolution of the background metric are ϱ , p and \mathcal{R} , and eq. (74) is replaced by a dispersion relation $p = p(\varrho)$, while in the case of scalar fields the relevant degrees of freedom are the scalar fields and \mathcal{R} , and the evolution is governed by the equations of motion of the scalar fields and the Friedmann equation. The explicit matter content and interactions determine the sequence and transitions of epochs in this approximation, and the present epoch of dust dominance $p \simeq 0$ outnumbers all previous epochs since expansion of the very hot and dense primeval plasma in its duration $\sim 10^{17}$ seconds. When solutions of the Friedmann equation for $p > -\varrho$ are evolved backwards in time we inavoidably hit an initial singularity $\varrho \rightarrow \infty$, $\mathcal{R} = 0$ for finite parameter t , and we will stick to the usual terminology of initial singularity or big bang, although we can only be sure that there existed a very hot and dense phase at some very early stage of our contemporary epoch¹¹. The Hubble parameter H_0 corresponding to the present value of $H(t)$ is still subject to a seminal debate among astronomers, and this is encoded in an uncertainty parameter h which varies between $0.5 < h < 0.85$ [76]:

$$H_0 = 100h \frac{\text{km}}{\text{Mpc} \cdot \text{s}} = 1.02h \times 10^{-10} \frac{1}{\text{yr}}.$$

It may be worthwhile to recall for the justification of (72) that the DMR experiment on COBE measured temperature anisotropies in the cosmic microwave background of order $\frac{\delta T}{T} \simeq 10^{-5}$ in multipole expansions up to $l = 30$, see [13, 117, 54] and references there, as well as [86] and [68] for compilations of measurements since COBE. The exciting result for the experts was the transition from an era of upper bounds on the anisotropy to actual measurements, but the results also show how good an approximation an isotropic universe represents up to the decoupling of the cosmic background radiation around 10^{12} seconds after the big bang¹².

Taking into account energy conservation (73), the Einstein equations reduce to

$$\dot{\mathcal{R}}^2 + k = \frac{\kappa}{3} \mathcal{R}^2 \varrho \quad (76)$$

and a flat universe would correspond to an averaged contemporary mass density

$$\varrho_c = \frac{3}{\kappa} H_0^2 = 1.9h^2 \times 10^{-26} \frac{\text{kg}}{\text{m}^3} = 81h^2 (\text{meV})^4 = 2.4h^2 \times 10^{-120} m_{Pl}^4.$$

Following the usual habit among astronomers energy densities will be measured in units of ϱ_c in this section: $\Omega = \frac{\varrho}{\varrho_c}$.

¹¹This qualification may seem strange, since our present investigation is mainly motivated from string theory. However, even within string theory we are currently in a phase of exploring more and more hitherto unknown (and unexpected) possibilities for the high energy sector of the theory, and yet there has not emerged a coherent proposal for the evolution of space-time near the Planck scale.

Besides this, quantum groups provide another set of challenging ideas about the shortest distance structure of space-time.

¹²Homogeneity is much more subtle from the experimental point of view, see the discussion in [15].

Particle physics influences (and is increasingly influenced by) cosmology in many ways. Two instances where scalar particles play major roles are inflation and dark matter:

Inflation denotes a phase of accelerated expansion of the universe when distances grew faster than light cones. This happens for pressure to density ratios between $-1 \leq \frac{p}{\varrho} < -\frac{1}{3}$, and temporary superluminal expansion has the potential to solve several major problems in cosmology, in particular the horizon problem, the negligibility of contributions to Ω from topological defects, and the problem why the measured energy density is not far away from ϱ_c . These and other motivations for inflation are very thoroughly reviewed in [15, 64, 79].

Although inflation seems to become an integral part of ongoing extensions of the standard cosmological model, I will not address it any further, since I do not expect that the dilaton which we examine here provides a suitable candidate for the sought for inflaton: We will see that thermally produced dilatons contribute at most 2 percent to the energy density of the universe at the scales where instantons and axions are expected to induce a mass term. On the other hand, this mass term also implies that the dilaton field can store energy in coherent oscillations which can provide a considerable amount of the contemporary energy density. Contrary to radiatively stored energy this energy would only very slowly dissipate into thermal energy of non-relativistic matter. In discussing this we will rely on the conservative assumption that at the time of onset of oscillations most energy is still stored in relativistic matter, i.e. we will calculate in a given Friedmann–Robertson–Walker background expanding according to $\mathcal{R} \sim \sqrt{t}$. Then the oscillations behave like non-relativistic matter with their energy density decreasing according to $\varrho \sim \mathcal{R}^{-3}$. In an alternative scenario one might speculate that the axion or dilaton could trigger temporary superluminal expansion of the universe with approximately constant energy density, if the axion and/or dilaton field would dominate the energy density immediately after the onset of oscillations. However, this would require a yet unknown mechanism to convert most of the thermal energy in relativistic matter into coherent axion or dilaton fields, and it would conflict with the successful interpretation of the cosmic microwave background as a remnant of the hot radiatively dominated phase before recombination if the energy is not re-converted into radiation.

Contrary to the verdict about dilaton induced inflation in FRW backgrounds, it turns out that a light dilaton can very well contribute to dark matter in the universe: There is wide agreement in the astronomy/astrophysics community that a considerable fraction of the contemporary energy density of the universe must be due to non-luminous matter, and the problem is to determine the nature of this matter. It seems clear now that part of this matter is of non-baryonic origin, since even for the lowest possible values of Deuterium abundance¹³ and a Hubble constant as low as $50 \frac{\text{km}}{\text{Mpc}\cdot\text{s}}$ primordial nucleosynthesis allows for a maximal baryonic contribution to the energy density of order $\Omega_B \leq 0.08$ [96], while both galactic motion and velocities on larger scales indicate values of Ω beyond 0.1. The recent survey of large scale peculiar motion in the nearby universe of Strauss and Willick [93] gives a range $0.3 \leq \Omega \leq 1$. Large scale motion, structure formation through gravitational

¹³Low D abundance means large conversion into ^4He due to large baryon density.

attraction, and gravitational lensing also indicate that a considerable amount of dark matter is concentrated in halos around galaxies or groups of galaxies, whence a large fraction of the dark matter has to be cold. For very massive particles this means that they had to be non-relativistic when they decoupled due to thinning out in the expanding universe. Very weakly interacting light particles contribute to cold dark matter (CDM) through non-relativistic coherent oscillations, as has been pointed out before and will be explained below. From the particle physics point of view the leading contenders for the CDM component of dark matter are the axion [64, 63], the lightest supersymmetric particle commonly denoted as a neutralino, and more recently the dilaton [50, 28, 34, 35, 49].

In this section we will focus on a dilaton whose mass at very low temperatures is dominated through instanton effects and discuss evolution of the axidilaton in an expanding universe. For this purpose we will concentrate on a temperature range between 1 TeV and 100 MeV, since we expect that a light axidilaton acquires its masses in that range (for $10^{11}\text{GeV} \leq f_\phi \leq 10^{18}\text{GeV}$) and it makes sense to assume that besides our hypothesized axidilaton no further degrees of freedom beyond the standard model will be relevant at these scales.

The universe has cooled down to temperatures 1 TeV and 100 MeV around $10^{-13} - 10^{-12}$ and $10^{-5} - 10^{-4}$ seconds after the initial singularity¹⁴. In this energy range the universe is radiation dominated with relativistic background matter satisfying a dispersion relation $\varrho = 3p$. The density and the scale factor then evolve according to

$$\varrho(t) = \frac{3}{4\kappa t^2} \frac{(\mathcal{R}_0^2 + kt_0^2)^2}{(\mathcal{R}_0^2 + kt_0(t_0 - t))^2}, \quad (77)$$

$$\mathcal{R}(t) = \sqrt{\frac{t}{t_0}} \sqrt{\mathcal{R}_0^2 + kt_0(t_0 - t)}, \quad (78)$$

where \mathcal{R}_0 is the scale parameter at a fixed time t_0 during radiation dominance. Evolving back the current energy density, which is within one order of magnitude of the critical density, shows that $\mathcal{R}_0 \gg kt_0$ during radiation dominance and curvature effects can be neglected. The energy density can then be estimated to relate time and temperature scales¹⁵:

$$\varrho = \frac{\pi^2}{30} g(T) T^4, \quad (79)$$

where $g(T)$ is the effective number of relativistic degrees of freedom in thermal equilibrium and varies by a factor of two for temperatures between 100 MeV and 1 TeV: If we assume standard model particle content plus an axidilaton we find in the high temperature regime

¹⁴The robustness of these scales against our ignorance of particle physics at very high energies is amazing: Eqs. (77,79) show that supersymmetry would divide these time scales only by a factor $\sqrt{2}$, and that one would need 10^4 additional relativistic degrees of freedom to invalidate the order of magnitude estimates!

¹⁵Without approximate flatness we would find a curvature term $-k\mathcal{R}^{-2}(P_i X^i)^2$ in the dispersion relation, and even the treatment of ideal gases would become very complicated.

$g(1\text{TeV}) = 105.75$ including the Higgs particle¹⁶ and the top quark, or $g(1\text{TeV}) = 94.25$ without them. In the low temperature sector we find $g(1\text{GeV}) = g(100\text{MeV}) = 49.75$, where the light particles included are the electron, up and down quarks, three left-handed neutrinos, the photon, eight gluons, the axion and the dilaton.

Thermally produced dilatons are relativistic, and their equilibrium density at temperature T is:

$$\nu(T) = \frac{\zeta(3)}{\pi^2} T^3.$$

From this and eqs. (77–79) we find a mild increase of the number of dilatons per comoving volume $v = V\mathcal{R}^{-3}$ with temperature:

$$N_\phi = \frac{\zeta(3)}{\pi^2} \left(\frac{45}{2g(T)} \right)^{3/4} \left(\frac{m_{Pl}}{\pi t_0} \right)^{3/2} \mathcal{R}_0^3.$$

This can easily be understood: As temperature approaches mass thresholds the effective number of relativistic degrees of freedom decreases and particles annihilate into the remaining light degrees of freedom.

We learn from $g(T)$ that thermally created dilatons contributed about 2% to the energy density of the universe for $1\text{ GeV} > T > 100\text{ MeV}$, if the dilaton was still in thermal equilibrium. On the other hand, we have seen that the dilaton is extremely weakly coupled, and it may well happen that it decouples from the heat bath at a temperature T_{dec} above 1 GeV. Then the energy density of dilatons which were thermally produced at the temperature T_{dec} is still governed by the temperature T of the heat bath as long as the heat bath remains relativistic. This holds for any massless decoupled particle species and is a simple consequence of the fact that the energy density of thermally produced decoupled species evolves according to $\varrho_{dec} \sim \mathcal{R}^{-4} \sim T^4$, i.e. the decoupled species cools out exactly like the relativistic heat bath. There is a difference, of course: The number of effective relativistic degrees of freedom seen by the decoupled particles was $g(T_{dec}) > g(T)$, and if thermally produced dilatons decoupled at T_{dec} their contribution to the energy density of the universe was $g(T_{dec})^{-1} < 2\%$, while their number density was reduced by a factor $(g(T)/g(T_{dec}))^{3/4}$. After photon recombination the contribution of thermal dilatons to Ω becomes negligible since $\varrho_{\phi,th}$ still decays with \mathcal{R}^{-4} , while the energy density in the dust decays only with \mathcal{R}^{-3} . Therefore, with or without decoupling thermally produced dilatons make no relevant contribution to the present energy density of the universe.

How then do the worries arise that the coupling scale of the dilaton is constrained to values below 10^{12} GeV from the requirement $\Omega \leq 1$, similar to the decay constant of the axion? This follows from the corresponding analysis for the axion in [80, 1, 37, 95], if f_ϕ is supposed to determine the expectation value $\langle \phi^2 \rangle$ of the dilaton at the onset of coherent oscillations. However, there is a caveat in this reasoning: It follows from the form of the instanton induced potential for the axion that its amplitude at the onset of oscillations is of the order f_a , since both a periodic and a non-periodic axion is at most $|\Delta a| \leq \pi f_a$ away from a local minimum of $V(a)$. On the other hand, such an estimate makes no sense for

¹⁶Right-handed neutrinos were excluded in the calculation of $g(T)$.

the dilaton since the low energy potential is not periodic and we have encoded any non-vanishing expectation value of the dilaton in our horizon in the gauge coupling. We also should not rely on dimensional arguments, since at the onset of oscillations there are two widely different mass scales which govern the dynamics of the dilaton: A very large decay constant and a very small mass.

As a consequence, we employ the relation (68)

$$m_\phi f_\phi \simeq \sqrt{6} m_a f_a$$

to estimate the contribution of dilaton oscillations to Ω . Coherent oscillations of the axion and the dilaton arise when the mass terms begin to dominate over the expansion terms in the equations of motion of scalars:

$$\ddot{\phi} + \frac{3}{2t}\dot{\phi} + m^2\phi = 0. \quad (80)$$

Classical trajectories of the axion and the dilaton satisfy this equation approximately, since due to the large decay constants the axion–dilaton coupling and the couplings to gauge fields provide negligible corrections to the linearized theory.

As long as mass terms can be neglected scalar fields approach stationary values with deviations fading with $t^{-1/2}$. On the other hand the field oscillates with frequency m if the mass term dominates, and this misalignment mechanism promotes light scalars to cold dark matter even though the temperature exceeds the mass. As a very heuristic argument to identify the transition region between the two regimes one may require smooth transition of $\dot{\phi}$. This gives for the transition time $2\tilde{t} = 3m^{-1}$. For times $t \gg m^{-1}$ the energy density $\varrho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2$ stored in the oscillations behaves exactly like pressureless matter under expansion: Dine and Fischler [37] pointed out that the constant mass solution to (80) is

$$\phi(t) = t^{-\frac{1}{4}}(A_+ J_{\frac{1}{4}}(mt) + A_- J_{-\frac{1}{4}}(mt)),$$

and the asymptotic expansion for $mt \gg 1$ implies $\varrho_\phi \sim \mathcal{R}^{-3}$. This scaling behavior of ϱ_ϕ persists in a dust dominated universe, where ϕ evolves with $t^{-1/2}J_{\pm\frac{1}{2}}(mt)$ and \mathcal{R} evolves with $t^{2/3}$, and the oscillations do not contribute to the pressure in the universe. Several other groups have analyzed the influence of the instanton induced axion mass and found that it begins to dominate over the expansion term at $T_a \simeq 1\text{GeV}$ [80, 1, 95].

A calculation of the temperature dependence of the axion mass by Gross, Pisarski and Yaffe was employed by Turner to determine the mass dependence of the temperature T_a beyond which the evolution of axions is dominated by oscillatory behavior [95]. He found that the transition temperature scales with m_a according to $T_a \sim m_a^{0.18}$, where m_a is the low temperature limit of the axion mass. Due to the constancy of the decay constants Turner's result also applies to the dilaton and we find for the corresponding scale

$$\frac{T_\phi}{T_a} \simeq \left(\frac{f_a}{f_\phi}\right)^{0.18},$$

whence the momenta of the oscillations entering the horizon are related by

$$\frac{p_\phi}{p_a} \simeq \frac{t_a}{t_\phi} \simeq \left(\frac{f_a}{f_\phi}\right)^{0.36}.$$

From these ratios follows an estimate on the velocity ratio, which can be used as a further indicator that coherent dilaton oscillations qualify as cold dark matter:

$$\frac{v_\phi}{v_a} \simeq \left(\frac{f_\phi}{f_a}\right)^{0.64}.$$

To discuss implications of the previous results on the role of the dilaton in cosmology we should distinguish two cases:

- S–duality applies at the GUT scale, implying that f_ϕ is bounded to be at most two orders of magnitude above the expected value for a misaligned axion $f_a \simeq 10^{12}$ GeV.
- S–duality is broken, with $f_a \simeq 10^{12}$ GeV but $f_\phi \simeq 10^{18}$ GeV.

The consequences in the first case are schematically similar to the consequences in the second case, but it leaves us with the puzzle to identify a mechanism which could lower f_ϕ by four orders of magnitude from its theoretically expected value. A further case very similar to the case of S–duality at the GUT scale would suppose S–duality at the QCD scale. A priori there seems no particular justification for this assumption, apart from the fact that it nicely complies with the mass estimate $m_\phi \geq 10^{-4}$ eV which would arise for a dilaton coupling to nucleon masses [43]. Then the dilaton decay constant would be close to the axion decay constant and the axion and the dilaton would develop oscillations at the same scale and make comparable contributions to Ω .

In the second case S–duality is maintained in the axidilaton sector, but not in the couplings to gauge fields. Eq. (68) then hints at a non–perturbatively generated dilaton mass which is much smaller than the axion mass:

$$m_\phi \sim 10^{-6} m_a$$

and the dilaton will start to oscillate after the axion, when the temperature has dropped by another factor of 10 and the time scale has expanded by two orders of magnitude.

The velocity of large scale fluctuations entering the horizon at the QCD scale is $v_\phi \sim 10^4 v_a$, and from $v_a \sim 10^{-6}$ [15] we learn that even in this sense dilaton oscillations remain non–relativistic for all choices of f_ϕ . Borrowing on the results of [80, 1, 37, 95] for the axion we find for the dilaton contribution to the contemporary energy density of the universe

$$\frac{\Omega_\phi}{\Omega_a} \simeq 10^{-5} \frac{\langle \phi^2 \rangle}{f_a^2} \simeq 10^7 \frac{\langle \phi^2 \rangle}{m_{Pl}^2}, \quad (81)$$

and the dilaton would make an appreciable contribution to the energy density for misalignment in the range

$$\frac{\sqrt{\langle \phi^2 \rangle}}{m_{Pl}} \sim 10^{-3} - 10^{-4}.$$

Taking into account the time evolution of the dilaton before instanton tunneling this corresponds to

$$\frac{\sqrt{\langle\phi^2\rangle}}{m_{Pl}} \simeq 1$$

for temperatures above 1 TeV, and the outlook for an appreciable fraction of dilatons in the dark matter seems promising. However, any further investigation of this subject requires better knowledge, or speculation, about new physics and evolution of a massless dilaton for temperatures above 1 TeV, and this is beyond the scope of the present work.

9 Conclusions and outlook

The appearance of fundamental scalars with a direct coupling to gauge curvature terms remains a challenge in string theory which offers unexpected rewards in low energy physics.

In order to resolve the ambiguity in the definition of gauge couplings in the presence of a massless dilaton, the dilaton has to acquire a mass at an early stage in the evolution of the universe. Motivated from the observation that both in string theory and in Kaluza–Klein theory the dilaton couples with different signs to axions and to gluons the proposal was made that rather than a gluino condensate it is a variance of the axion in the Euclidean domain which stabilizes the dilaton. For consistency this proposal has to rely on the assumption that the four–dimensional field theory containing axions and dilatons is an effective theory, with topological defects stabilized through higher derivative terms, as is the case e.g. in string theory. An S–dual coupling between the axion and the dilaton then yields an estimate on the dilaton mass $m_\phi \simeq m_a f_a f_\phi^{-1}$. Comparison with the simplest supersymmetric extension of an axidilaton–gluon theory revealed that the axion coupling should dominate the dilaton mass for decay constants $f_\phi > 10^{11}$ GeV and gluino condensates below 1 TeV³.

We have pointed out that the dynamics of a light scalar in an expanding universe before mass dominance easily accomodates for large coupling scales without overclosing the universe as long as no multivalued vacua emerge. For a dilaton with coupling scale $f_\phi \simeq m_{Pl}$ this means that a variance $\sqrt{\langle \phi^2 \rangle} \simeq m_{Pl}$ is still permissible at a temperature $\simeq 10^3 T_\phi$, where T_ϕ is the temperature where coherent dilaton oscillations evolve. In this case we have seen that the dilaton provides an interesting candidate for cold dark matter accompanying an axionic component, and for a coupling to QCD we found an estimate $T_\phi \simeq 10^{-1} T_a$. As a consequence the onset of dilaton oscillations seems to be close to or even coincide with the QCD phase transition.

We have also seen that a dilaton coupling to gauge curvature terms provides a simple mechanism to accomodate both a regularized Coulomb potential and a confining potential in gauge theory. Given this observation and the fact that string theory unavoidably predicts a dilaton coupling to gauge fields, continuing investigation of light dilatonic degrees of freedom seems more than justified. It is of particular interest to see how the transition from the confining solution to a regularized Coulomb potential proceeds, and which parameters control the phases of a gauge theory coupling to a dilaton.

In conclusion, gauge theories with a dilaton present rather an interesting than a worrisome prediction of string theory.

Appendix A: Conventions and notation

We use Greek letters for three- and four-dimensional holonomic indices, while anholonomic indices are denoted by Latin letters from the beginning of the alphabet. Higher dimensional holonomic tangent frame indices are denoted by capital letters from the middle of the alphabet and anholonomic indices by capital letters from the beginning of the alphabet. Hence, components of the 4-bein and the D -bein in $D > 4$ dimensions read e_μ^a , E_M^A . Latin letters from the middle of the alphabet are used both for Lie algebra indices and for spatial Minkowski space indices in $3 + 1$ dimensions, and matrix elements of Lie algebra generators are written as $(X_i)_{ab}$. We use a boldface notation for 3-vectors. Gauge couplings are usually denoted by q , while g is reserved for the metric in three or four dimensions.

Our conventions for Planck units are rescaled by a factor $\sqrt{8\pi}$:

$$m_{Pl} = \kappa^{-1/2} = (8\pi G)^{-1/2} = 2.4 \times 10^{18} \text{ GeV},$$

$$t_{Pl} = 2.7 \times 10^{-43} \text{ s},$$

$$l_{Pl} = 8.1 \times 10^{-35} \text{ m}.$$

In the literature these units are sometimes referred to as reduced Planck units.

The generic setting for quantum field theory are total spaces fibered by a usually highly reducible representation space of a group $SO(1, D-1) \times G$. G is referred to as a gauge group, and is assumed to be a compact Lie group consisting of simple factors. It is generated by a Lie algebra with relations

$$[X_i, X_j] = if_{ij}^k X_k.$$

The fiber projects down to a D -dimensional base space \mathcal{M} of Minkowski signature $(-, +, \dots, +)$, and $SO(1, D-1)$ is the structure group of the tangent bundle. The generators of $SO(1, D-1)$ as well as their representations are denote by $L_{ab} = -L_{ba}$.

Covariant derivatives and curvatures are defined via

$$D_\mu = \partial_\mu + \omega_\mu - iqA_\mu = \partial_\mu - \frac{1}{2}\omega_{b\mu}^a L_a^b - iqA_\mu^j X_j,$$

$$R_{\mu\nu} - iqF_{\mu\nu} = -\frac{1}{2}R_{b\mu\nu}^a L_a^b - iqF_{\mu\nu}^j X_j = [D_\mu, D_\nu],$$

and the dual curvature tensor \tilde{F} in four dimensions is

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}.$$

F is denoted as self-dual in Minkowski space if

$$\tilde{F}_{\mu\nu} = iF_{\mu\nu}.$$

A Wick rotation maps the Schrödinger equation into the diffusion equation through¹⁷

$$t \rightarrow -i\tau$$

and the transition from Minkowski space field theory to Euclidean field theory proceeds via

$$\begin{aligned}\varphi(t, \mathbf{x}) &\rightarrow i^{\#(0)} \varphi_E(\tau, \mathbf{x}), \\ \mathcal{L}(\varphi) &\rightarrow -\mathcal{L}_E(i^{\#(0)} \varphi_E),\end{aligned}$$

where $\#(0)$ denotes the net number of covariant timelike indices in the field φ . The ϵ -tensor is not covariantly transformed under Wick rotation and satisfies $\epsilon^{0123} = \frac{1}{\sqrt{|g|}}$, thus accounting for the oscillatory instanton contribution through the axion–gluon coupling.

On the level of partition functions

$$Z[J] = \exp(iW[J]) = \int D\varphi \exp(iS[\varphi] + i \int d^4x J \cdot \varphi)$$

is mapped to

$$Z_E[J_E] = \exp(-W_E[J_E]) = \int D\varphi_E \exp(-S_E[\varphi_E] + \int d^4x J_E \cdot \varphi_E).$$

The mean fields are

$$\begin{aligned}\phi(x) &= \frac{\delta W[J]}{\delta J(x)}, \\ \phi_E(x) &= -\frac{\delta W_E[J_E]}{\delta J_E(x)},\end{aligned}$$

and the effective actions are accordingly

$$\begin{aligned}\Gamma[\phi] &= W[J] - \int d^4x J(x) \cdot \phi(x), \\ \Gamma_E[\phi_E] &= W_E[J_E] + \int d^4x J_E(x) \cdot \phi_E(x).\end{aligned}$$

The effective actions and the mean fields thus encode the quantum dynamics of the system under consideration in terms of classical evolution equations:

$$\begin{aligned}\frac{\delta \Gamma[\phi]}{\delta \phi(x)} &= -J(x), \\ \frac{\delta \Gamma_E[\phi_E]}{\delta \phi_E(x)} &= J_E(x).\end{aligned}$$

Note that under Wick rotation $\delta(x) \rightarrow i\delta(x)$, and therefore $J(t, \mathbf{x}) \rightarrow (-i)^{\#(0)} J_E(\tau, \mathbf{x})$, but

$$\frac{\delta}{\delta J(x)} \rightarrow i^{\#(0)+1} \frac{\delta}{\delta J_E(x)}.$$

The mean fields thus transform like the quantum fields: $\phi(x) \rightarrow i^{\#(0)} \phi_E(x)$.

¹⁷van Nieuwenhuizen and Waldron recently pointed out that the Wick rotation has a continuous extension in terms of a five-dimensional Lorentz-boost. They employed this observation to identify the action of the Wick rotation on spinors [74].

Appendix B: A remark on perturbative aspects of the axidilaton

We have argued from the Kaluza–Klein type coupling of a dilaton to gauge fields that instantons should provide a mechanism to stabilize a dilaton in four dimensions, irrespective from the presence or absence of higher massive modes. It is tempting to push this argument a little further and conclude that no effective potential should be generated perturbatively: In a low–dimensional Kaluza–Klein theory the volume of internal dimensions effectively only rescales the higher–dimensional Planck mass and axion constant to their low–dimensional values, and we have seen that the axion and dilaton couplings carry no further remembrance of the compactification scale. Thus in the low energy approximation the compactification scale reduces to a mass threshold, but there is no traceable imprint of this scale in the low energy sector¹⁸. Hence perturbation theory in the low energy sector can not reveal the presence of a compactification scale or its actual value, and therefore it can not remove the degeneracy of the dilaton. The same conclusion then should apply to any theory with a dilaton as long as only Kaluza–Klein type couplings are considered. The shaky point about this argument concerns the non–renormalizability of the model under discussion and the question of the very meaning of perturbation theory. One is on much safer ground if supersymmetry can be employed to exclude a perturbatively generated dilaton potential [39], but going below the SUSY scale we have left that safe harbor behind. Nevertheless, it turns out that the reasoning is not in contradiction with a 1–loop calculation as long as one relies on dimensional regularization:

On the 1–loop level the effective potential is generated by 1–loop diagrams with only axions and dilatons of vanishing momenta as external particles. There appear three types of relevant tree level vertices in (52):

— For external axions with gauge bosons in the loop there is only one relevant vertex:

$$i\delta_{jk}\frac{q^2}{8\pi^2 f_a}\epsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma, \quad (82)$$

where the gauge bosons at the vertex have momenta p_1, p_2 , polarizations μ, ν and orientations j, k in the Lie algebra. We have also taken out one factor of i into the momentum conserving factor $(2\pi)^4 i\delta(p_1 + p_2 + k)$, with k denoting the 4–momentum of the axion.

— For external dilatons with gauge bosons in the loop there are enumerably many vertices:

$$\delta_{jk}\frac{1}{(if_\phi)^n}(p_{1\nu}p_{2\mu} - \eta_{\mu\nu}p_1 \cdot p_2). \quad (83)$$

— The corresponding axion–dilaton vertices are

$$-\left(\frac{2i}{f_\phi}\right)^n p_1 \cdot p_2, \quad (84)$$

¹⁸Of course, in a real Kaluza–Klein system there would be scalars in the adjoint representation of the gauge group, and with chiral fermions one could infer the likely existence of a compactification scale, but one would not have any hint for its order of magnitude.

where now p_1 and p_2 are the incoming axion momenta.

From the vertices it is immediately clear that no perturbatively generated axion potential appears in the theory: (84) vanishes due to the vanishing axion momenta, while (82) vanishes due to momentum conservation with vanishing external axion momentum.

On the other hand, a diagram with external zero-momentum dilatons and axions or gluons in the loop is directly proportional to $\int d^4p$, and this vanishes in dimensional regularization [100]. After Wick rotation one finds

$$\int d^{4-2\varepsilon}p = \lim_{\alpha \rightarrow 0} \int d^{4-2\varepsilon}p \frac{1}{(p^2 + m^2)^\alpha} = \lim_{\alpha \rightarrow 0} i\pi^{2-\varepsilon} m^{4-2\varepsilon-2\alpha} \frac{\Gamma(\alpha - 2 - \varepsilon)}{\Gamma(\alpha)} = 0$$

and no ϕ^n -vertices could be inferred from these diagrams.

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